FIRST·BOOKS·OF·SCIENCE

# PRACTICAL·MATHEMATICS

USHERWOOD·&·TRIMBLE

3 1761 05478156 2

*FIRST BOOKS OF SCIENCE*

# A FIRST BOOK OF PRACTICAL MATHEMATICS

MACMILLAN AND CO., LIMITED
LONDON · BOMBAY · CALCUTTA
MELBOURNE

THE MACMILLAN COMPANY
NEW YORK · BOSTON · CHICAGO
DALLAS · SAN FRANCISCO

THE MACMILLAN CO. OF CANADA, LTD.
TORONTO

Mat
U855f

# A FIRST BOOK OF PRACTICAL MATHEMATICS

UNIVERSITY OF TORONTO LIBRARY

BY

T. S. USHERWOOD

B.SC., WH.EX., A.M.I.MECH.E.
HEAD OF MANUAL TRAINING DEPARTMENT, CHRIST'S HOSPITAL

AND

C. J. A. TRIMBLE

B.A., LATE SCHOLAR TRIN. COLL., CAMB.
MATHEMATICAL MASTER AT CHRIST'S HOSPITAL

145937
3 | 5 | 18

MACMILLAN AND CO., LIMITED
ST. MARTIN'S STREET, LONDON
1913

COPYRIGHT

# PREFACE

In preparing this introduction to Practical Mathematics, the authors have dealt only with those parts of the subject which seem to them to be of real value in practical work. They desire to thank Prof. R. A. Gregory and Mr. A. T. Simmons for valuable assistance given them during the preparation of the book and its passage through the press. Acknowledgments are also due to the Union of Lancashire and Cheshire Institutes (U.L.C.I.), the Midland Counties Union of Educational Institutions (M.C.U.), the Board of Education (B.E.), London University (Matric.), the Civil Service Commissioners (C.S.C.), and other authorities, who have kindly permitted the use of questions from their examination papers.

T. S. U.
C. J. A. T.

Christ's Hospital,
*August*, 1913.

Digitized by the Internet Archive
in 2007 with funding from
Microsoft Corporation

http://www.archive.org/details/firstbookofpract00usheuoft

# CONTENTS

CHAPTER | PAGE

I. THE USE OF SIGNS, SYMBOLS AND ABBREVIATIONS. POSITIVE AND NEGATIVE QUANTITIES - - 1

II. THE DECIMAL NOTATION. THE INDEX NOTATION. THE METRIC SYSTEM. WEIGHTS AND DENSITIES 17

III. SIGNIFICANT FIGURES. ROUGH CHECKS ON ACCURACY. CONTRACTED METHODS OF CALCULATION 32

IV. ELEMENTARY MENSURATION - - - - 54

V. THE USE OF FORMULAE. ALGEBRAIC MANIPULATION. EQUATIONS - - - - - - 72

VI. THE USE OF SQUARED PAPER. GRAPHS - - 86

VII. FACTORS. H.C.F. AND L.C.M. FRACTIONS - 113

VIII. ALGEBRAIC MANIPULATION. EXPANSION AND FACTORISATION - - - - - - - 128

IX. AVERAGES, PERCENTAGES. RATIO AND PROPORTION - - - - - - - - - 138

X. MENSURATION OF CIRCLE, CYLINDER AND CONE - 158

TYPICAL EXAMINATION PAPERS - - - - 164

ANSWERS TO EXERCISES - - - - - 171

INDEX - - - - - - - - 183

# PRACTICAL MATHEMATICS.

## CHAPTER I.

### THE USE OF SIGNS. SYMBOLS AND ABBREVIATIONS. POSITIVE AND NEGATIVE QUANTITIES.

IN the following pages, a knowledge of Elementary Arithmetic has been assumed, but it has been considered advisable to discuss the application of the four simple rules of arithmetic to numbers in which decimals occur. In order to economise time and space, the definitions and methods of using certain signs and symbols are given in this chapter.

**The Sign of Addition.**—In addition the sign + (read **plus**) is used. Thus $4+7$, which is read 'four plus seven,' means 'to 4 add 7.'

$8+11$ means 'to 8 add 11.'

$7\frac{1}{2}+4\frac{3}{4}$ means 'to $7\frac{1}{2}$ add $4\frac{3}{4}$.'

$a+b$ means 'to $a$ add $b$,' where $a$ and $b$ are any two numbers whatever.

The **expression** $4+7$ equals 11, but the expression $a+b$ cannot be simplified unless the numerical values of $a$ and $b$ are known.

Thus, if a flagstaff 12 ft. long be erected on the top of a house 45 ft. high, the top of the pole is $12+45$, that is, 57 ft. from the ground. But if a flagstaff 3 yd. long stands on the top of a house 45 ft. high, the top of the pole is 3 yd. + 45 ft. from the ground, and the expression 3 yd. + 45 ft. cannot be simplified unless we know how many feet there are in a yard.

If a wire $a$ inches long is soldered to a wire $b$ inches long, the overall length may be written as $a+b$ inches or as $b+a$ inches; but these expressions cannot be simplified unless the numerical values of $a$ and $b$ are given.

In the same way, $a+b+x$ means 'to $a$ add $b$ and to the result add $x$.'

$r_1+r_2+r_3$ means 'add $r_1$, $r_2$, $r_3$ together.'

The value of $a+b+x$ is called the **sum** of $a$, $b$ and $x$.

The result of adding $r_1$, $r_2$ and $r_3$ is called the **sum** of $r_1$, $r_2$ and $r_3$.

Thus, the sum of 3, $4\frac{1}{2}$, $5\frac{1}{4}$ is $3+4\frac{1}{2}+5\frac{1}{4}$; that is, $12\frac{3}{4}$.

The sum of £2. 13*s.* 6*d.*; 5*s.* 9*d.*; 3*s.* 1*d.*; and £1. 0*s.* $10\frac{1}{2}$*d.* is £2. 13*s.* 6*d.* + £0. 5*s.* 9*d.* + £0. 3*s.* 1*d.* + £1. 0*s.* $10\frac{1}{2}$*d.*; that is, £4. 3*s.* $2\frac{1}{2}$*d.*

The joint resistance, to the flow of an electric current, of a number of resistances $r_1$, $r_2$, $r_3$, $r_4$, etc., in series is their sum; that is,
$$r_1+r_2+r_3+r_4+etc.$$

If $w$ lb. of molten iron is run into a crucible weighing W lb., the total load on the hook supporting the crucible is the sum of $w$ and W; that is, $w+\mathrm{W}$ lb.

## EXERCISES I.

**1.** Find £2. 10*s.* 0*d.* + 3*s.* 5*d.* + £7. 6*s.* $2\frac{1}{2}$*d.*

**2.** A farmer's stock consists of 12 cows, 4 horses, 100 sheep and 30 pigs. Give an expression for his total stock, using $c$ to denote cows; $h$, horses; $s$, sheep; and $p$, pigs. How many animals had he?

**3.** Find the value of $a+b+c+d$ when $a$ is 2; $b$, $1\frac{1}{2}$; $c$, $2\frac{1}{4}$; and $d$, $5\frac{3}{4}$.

**4.** A bucket weighing $w$ lb. is used to draw water from a well; it is supported by a chain weighing $p$ lb. The bucket holds W lb. of water. What total weight is lifted in drawing a bucket full of water?

**5.** The following resistances are arranged in series: $r_1$, R, $r_2$, R′. Write down the joint resistance. What is the joint resistance if $r_1$ is $2\frac{1}{2}$, R is $3\frac{1}{4}$, $r_2$ is 20 and R′ is 5?

**6.** A beam of weight W lb. is supported at both ends and carries loads of $\mathrm{W}_1$, $\mathrm{W}_2$ and $\mathrm{W}_3$ lb. What is the total load on the two supports?

**7.** Find the value of

$$x+y+z, \quad y+x+z, \quad z+y+x \quad \text{and} \quad x+z+y$$

(*a*) When $x$ is 0; $y$, 4; and $z$, $2\frac{1}{2}$.
(*b*) When $x$ is 40; $y$, 27; and $z$, $16\frac{1}{2}$.
(*c*) When $x$ is $13\frac{1}{2}$; $y$, $2\frac{3}{4}$; and $z$, $\frac{1}{2}$.

**8.** Evaluate, that is, find the value of

$2x+3y+z+4\frac{1}{2}w$, $z+2x+4\frac{1}{2}w+3y$, and $4\frac{1}{2}w+3y+2x+z$

(*a*) When $x$ is $3\frac{1}{2}$ ; $y$, 1 ; $z$, 10 ; and $w$, 8.
(*b*) When $x$ is 2 ; $y$, 2 ; $z$, 2 ; and $w$, 2.
(*c*) When $x$ is 0 ; $y$, 5 ; $z$, 5 ; and $w$, 5.

**9.** Add together $2x$, $6x$, $3y$ and $2z$.

**10.** Find the sum of $4x+y+5z$, $3y+z+2x$ and $3z+3x+y$.

**11.** The length of a room is L yards and its breadth is B yards. What is its perimeter, that is, the distance round the room? If the room is H yards high, what is the perimeter of (*a*) a side wall ; (*b*) an end wall?

**The Sign of Subtraction.**—For subtraction the sign − (read **minus**) is used.

Thus, 7 − 4 means 'from 7 take 4.'

11 − 8 means 'from 11 take 8.'

$7\frac{1}{2}-4\frac{3}{4}$ means 'from $7\frac{1}{2}$ take $4\frac{3}{4}$,' and so on.

So that $a-b$ means 'from $a$ take $b$,' where $a$ and $b$ are any two numbers whatever, $a$ being greater than $b$. The expression $a-b$ is called the **difference** of $a$ and $b$.

It would clearly be a convenience to be able to translate $a-b$ as meaning 'from $a$ take $b$,' where $a$ and $b$ are any two numbers whatever without restriction as to their magnitude.

We have to be able to interpret such results as 4 − 7, $4\frac{3}{4}-7\frac{1}{2}$, etc.

Consider the series of operations involved in the following example. 'From $a$ take $b$ and to the result add $b$.' The final result is evidently $a$, for by the second operation we restore to $a$ what was removed by the first operation.

Symbolically, we write $a-b+b$ is the same as $a$.

Thus the numbers $-b$, $+b$, as it were, neutralise one another: the sign −, then, denotes the **inverse** of the sign +.

If a man has no money and owes £6, we may say that his liabilities are £6, his assets are −£6, assets being the inverse of liabilities. The statement 'his assets are −£6' means that his assets are less than nothing by £6 : that is to say, he must pay away £6 from his future earnings, or diminish his future earnings by £6 before he has any assets.

If a man has £8 and owes £3, his net assets are £5; his liabilities are −£5 (less than nothing by £5). Any future liabilities may be met to the extent of £5 before he runs into debt.

If a man has £7 and owes £11, his net assets are −£4; his liabilities are +£4.

If a certain town be 30 miles due North of London, we may express the fact by saying that it is −30 miles due South of London, distances North of a certain point being inverse of distances South of that point.

If a man walk 4 miles due East, then 2 miles due West, then 7 miles due East again, he is finally 9 miles due East of his starting point: or, −9 miles due West of it.

Using signs, we may say:

Final distance from starting point is $4E - 2E + 7E$, that is, $9E$, where $E$ stands for East. The diagram in Fig. 1 illustrates this.

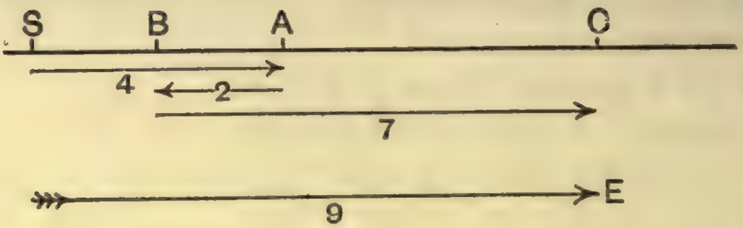


FIG. 1.

Ex. 1. *A man has* £14 *and owes* £5. 10*s*. 0*d*. *He spends* £4. 10*s*. 6*d*. *What are his assets?*

His assets are £14 − £5. 10*s*. 0*d*. − £4. 10*s*. 6*d*., that is, £3. 19*s*. 6*d*.

Ex. 2. *Add* 14 x's *to* 5 x's *and subtract* 9 x's *from their sum.*

Result is $14x + 5x - 9x$, that is, $19x - 9x$ or $10x$.

It should be noticed that the difference of two numbers is unaltered if each is increased by the same amount. Thus, $7 - 4$ is the same as $10 - 7$, where each number is increased by 3.

Similarly, $a - b$ gives the same result as that obtained by subtracting $b + x$ from $a + x$.

If, commencing with any number, say 6, a series of numbers is written down, each being one less than the preceding, we have,

6 5 4 3 2 1.

The next number would be $1 - 1$, that is, 0.

In order to be able to continue the series beyond 0 we invent the symbols $-1$, $-2$, $-3$, etc., where $-1$ means $0-1$, $-2$ means $-1-1$, and so on.

**Positive and Negative Numbers.**—Any number preceded by a minus sign is called a **negative** number. If there is no sign, or the number is preceded by a plus sign, it is a **positive** number.

Thus, if a displacement to the right of a certain point is regarded as a positive displacement, one to the left will be regarded as negative. If motion due North is positive, then motion due South is negative.

The dates of events which occurred B.C. may be said to be − A.D.

Heights of mountains and depths of seas are measured from sea level. It is correct, although unusual, to say that the depth of Mount Everest is about − 29,000 ft.

If compressive forces are regarded as positive, tensile forces will be regarded as negative ; and *vice versâ*.

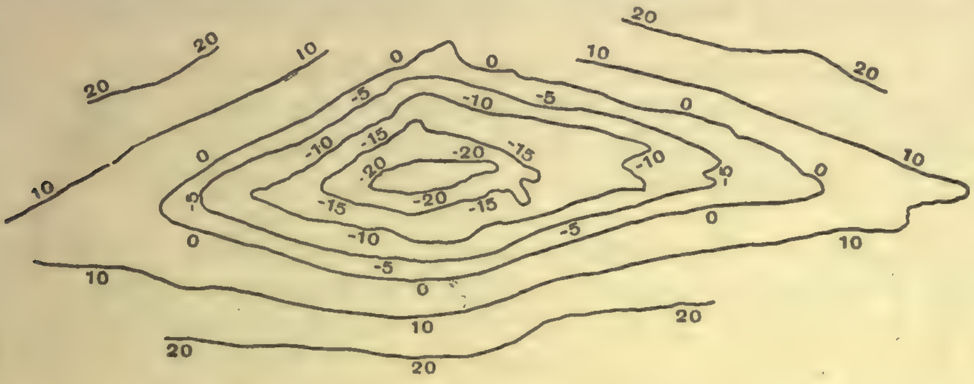


FIG. 2.

In contour maps, heights are denoted by positive and depths by negative numbers. Fig. 2 shews part of a contour map : the whole of the area in the figure within the line marked 0 is below sea level ; the area shewn outside that line is above sea level.

## EXERCISES II.

**1.** If motion upwards be positive, what is motion downwards?

**2.** If motion downwards be positive, what is motion upwards?

**3.** If assets be regarded as positive, what will liabilities be regarded as?

**4.** On the Fahrenheit scale the freezing point of water is marked 32 degrees. What is the temperature of a body which is (*a*) 70 degrees above freezing point; (*b*) 10 degrees below freezing point; (*c*) 70 degrees below freezing point?

*N.B.*—The symbol ° is used for 'degrees.' Thus, 212° is read '212 degrees.'

**5.** What is the temperature of a body which is $x^\circ$ above freezing point on the Fahrenheit scale? What is its temperature if it is $t^\circ$ below freezing point?

**6.** The boiling point on the Fahrenheit scale is marked 212°. What is a temperature of (*a*) 72° below boiling point; (*b*) 40° above boiling point; (*c*) $t^\circ$ below boiling point?

**7.** On the Centigrade scale freezing point is marked 0° and boiling point is marked 100°. What is a temperature of 273° below freezing point? How far is this point below boiling point? How far is it *above* freezing point?

**8.** If temperatures are to be measured from a point 273° below freezing point on the Centigrade scale (*i.e.* if this point be marked 0° and temperatures be measured from it), what is the temperature of a body which is $t^\circ$ on the Centigrade scale?

**9.** Express with positive signs: a displacement of $-s$ feet vertically downwards; $-800$ feet below sea level; $-200$ A.D.; $-400$ B.C.; a speed of $-40$ feet per second North-West; a speed of $-50$ miles per hour to the West.

**10.** Express with negative signs: a mine 400 yards deep; $+950$ feet above sea level; 820 feet below sea level; a displacement of $y$ feet to the North; a speed of $v$ feet per second due East; 200 B.C.; a temperature of 100° above zero; a temperature of 57° below zero.

Find the value of each of the following:

**11.** £7. 8*s*. 4*d*. + £8. 16*s*. 7*d*. − £12. 9*s*. 3*d*.

**12.** $7\frac{1}{2}+8\frac{1}{4}-13\frac{3}{4}-7+3$.

**13.** 2*s*. 6*d*. − 3*s*. + 5*s*. 8*d*. − 10*s*. 6*a*.

**14.** $11-4-7+14-18$.

**15.** 7 cows − 2 cows.

**16.** 4 pounds − 5 pounds.

**17.** 18 $a$'s + 23 $a$'s − 14 $a$'s.

**18.** 2 $a$'s − 5 $a$'s.

**19.** $12x-20x$.

**20.** $7y-3y+8y-10y$.

Answer the following questions:

**21.** The perimeter* of a room is $2p$ feet. The room is $a$ feet long; what is its breadth?

**22.** An electrical circuit consists of three resistances in series. The joint resistance is R ohms and two of the included resistances are known to be $r_1$ and $r_2$ ohms respectively. What is the third resistance?

**23.** A bowl filled with water weighs W lb. When empty it weighs $w$ lb. What weight of water does the bowl contain?

**24.** A beam weighing W lb. is supported at its ends. The pressure on one end is $W_1$ lb. and on the other end is $W_2$ lb. What load does the beam carry?

**The Sign of Multiplication.**—In multiplication the signs × or . (read **multiplied by** or **times**) are used. When the dot is used as a sign of multiplication, it is usually printed at the bottom of the line to distinguish it from the decimal point, which is placed in a higher position.

Thus, $2 \times 3$ or $2 . 3$ means '2 times 3.'

$12 \times 2\frac{3}{4}$ or $12 . 2\frac{3}{4}$ means '12 times $2\frac{3}{4}$.'

$a \times b$, or $a . b$, or $ab$ means '$a$ times $b$.'

The expression $ab$, where $a$ and $b$ may be any numbers, is called the **product** of $a$ and $b$. This notation must not be confused with similar expressions occurring in arithmetic. Thus 73 means seventy-three or $7 \times 10 + 3$; but $ab$ does *not* mean $a$-ty $b$ or $a \times 10 + b$. Again $11\frac{3}{4}$ means $11 + \frac{3}{4}$, whereas $ab$ does *not* mean $a + b$.

In the same way $abc$ means '$a$ times $b$ times $c$,' and is called the product of $a$, $b$ and $c$. It is clear that $abc$, $bca$, $cab$, $cba$, etc., mean the same thing.

Thus, $16 \times \frac{1}{4}$ is the same as $\frac{1}{4} \times 16$, that is, 4.

$\frac{2}{3} \times \frac{1}{2}$ is the same as $\frac{1}{2} \times \frac{2}{3}$, or $\frac{1}{3}$.

$2 \times 3 \times 5$, $3 \times 5 \times 2$, $2 \times 5 \times 3$ all have the same value, namely 30.

It must not be assumed that multiplication necessarily involves increase, as might be inferred from such phrases as 'increase and multiply,' 'multiplies itself rapidly.'

---

* See question 11, p. 3.

Thus, $3 \times \frac{1}{2}$ means 'three times one-half,' that is, three halves, or $1\frac{1}{2}$, which is less than 3.

$\frac{1}{2} \times \frac{1}{2}$ is $\frac{1}{4}$, which is less than $\frac{1}{2}$.

$20 \times \frac{4}{5}$ is $\frac{4}{5}$ of 20 or 16, which is less than 20.

**Multiplication as Continued Addition.**—Multiplication was originally a method of doing continued addition.

Thus, $3+3+3$ is equivalent to $3 \times 3$.

$7+7+7+7$ may be written $4 \times 7$.

$b+b+b+b+\ldots+b$, where there are $a$ $b$'s, is the same as $ab$, $a$ being a whole number.

Ex. 1. *Fig. 3 shews a row of rivets, pitch $2\frac{1}{2}$ inches (that is, the distance between any consecutive pair of rivets is $2\frac{1}{2}$ inches). What is the distance from the centre of rivet* A *to the centre of rivet* B?

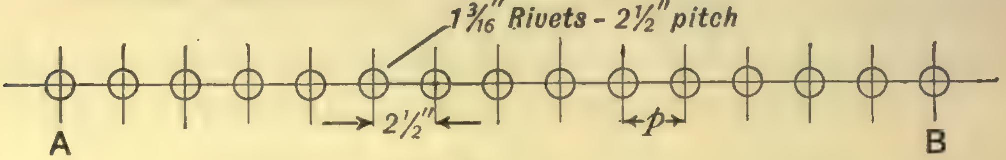


FIG. 3.

There are 14 spaces, each $2\frac{1}{2}$ inches wide.

Therefore the distance AB is $14 \times 2\frac{1}{2}$ or 35 inches.

Ex. 2. *If the pitch in the last example is* p *inches, and there are* n *spaces, what is the distance from centre to centre of the end rivets?*

It is np inches.

Ex. 3. *If there are* n *rivets and the pitch is* p *inches, what is the overall distance from centre to centre of the end rivets?*

The number of spaces is one less than the number of rivets. Compare Fig. 3.

Therefore the number of spaces is $n-1$.

The required distance in inches is, therefore, the product of $n-1$ and p. This is written $(n-1)p$ or $p(n-1)$.

The sign ( ) is called a **bracket** (see p. 82).

Remember that $p(n-1)$ means 'subtract 1 from n and multiply the result by p.'

Ex. 4. *An oak fence is made of* 3 *inch palings with* 2 *inch spaces between each. There are* 24 *palings between a pair of posts and the posts are* 9 *inches wide. Find the distance from centre to centre of the posts.*

There are 24 palings, and, therefore, 25 spaces 2 inches wide. The required distance in inches is

$24 \times 3$ (for palings) $+ 25 \times 2$ (for spaces) $+ 4\frac{1}{2} + 4\frac{1}{2}$ (for posts).

The distance is $72 + 50 + 9$ or 131 inches.

Ex. 5. *Evaluate* $(2+3)(6-4)$.

The value of the expression in the first bracket is 5, and of that in the second is 2.

Therefore $(2+3)(6-4)$ means $5 \times 2$, that is, 10.

Ex. 6. *What is the meaning of* $(a-1)(a+1)$?

The expression means 'subtract 1 from a, add 1 to a and multiply the two results.'

## EXERCISES III.

**1.** Express in words $a(b+c)$, $x(1-z)$, $(p+3)(y-5)$.

**2.** If the pitch of a single threaded screw (that is, the distance from any point on one turn of the thread to the corresponding point on the next turn) be $p$ inches and the number of turns be $n$, what is the overall length of the screw?

**3.** A beam carries a load of $w$ lb. per foot run (that is, carries $w$ lb. distributed over *each* foot of its length) and has a span of $l$ feet. What is the total load it carries?

**4.** In winding the field magnets of a dynamo there are $n$ turns each containing $l$ centimetres of wire. How would you find the total length of wire required?

**5.** The resistance of a wire is found by multiplying the resistance of 1 foot by its length in feet. The resistance of 1 foot of a certain copper wire is $r$ ohms. What is the resistance of 200 turns of this wire each $3\frac{1}{10}$ feet long?

**6.** Each rivet employed on a certain girder weighs $w$ lb. The span of the girder is $l$ feet and there are $n$ rivets in each foot of the span. How would you find the total weight of rivets required for the girder?

**7.** If the frontage of a house is 30 feet, there are 25 such houses in a terrace and there are no spaces between the houses, what is the length of the terrace?

**8.** Express symbolically the distance between the centre lines of the posts in the fence shewn in Fig. 4.

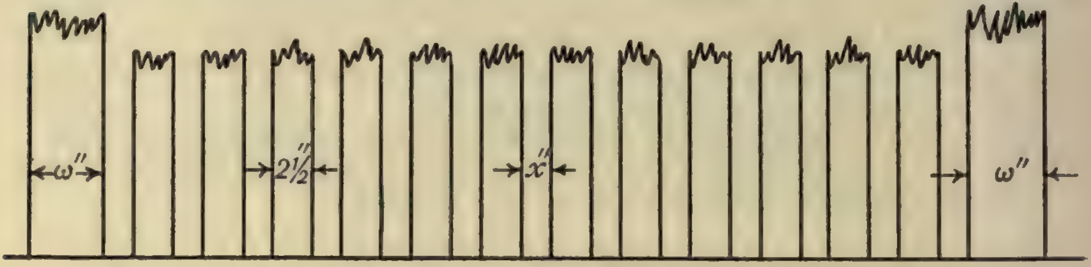


FIG. 4.

**9.** A workman has no scale and wishes to find the length of a wall. He knows that a brick is 9 inches long and counts $n$ bricks in a 'stretcher,' that is, a lengthways course. How long is the wall?

*N.B.*—The mortar joint is neglected. One might allow $\frac{1}{8}$ of an inch for each joint in good brickwork or from $\frac{1}{4}$ to $\frac{3}{8}$ of an inch in common brickwork.

**10.** A straight passage is paved with tiles 7 inches square (that is, 7 inches wide and 7 inches long). How would you find the length and breadth of the passage if you had no scale? Assume there are $9\frac{1}{2}$ tiles in a row from wall to wall and 156 tiles in a row from end to end.

**11.** A girder 12 feet long is supported at its ends and carries loads of W lb. at intervals of 2 feet. The girder weighs $w$ lb. What is the total load supported?

Assuming that the pressure on each support is the same, find this pressure when $w$ is 120 and W is 50.

**12.** Concrete consists of a mixture of cement, fine gravel and coarse gravel. For walls, 1 bag of cement is mixed with 3 bags each of fine and coarse gravel. In making a certain wall, $n$ bags of cement were used each occupying $v$ cubic feet. How many bags of fine and coarse gravel are required? Assume that there is no contraction on mixing and find the volume of the wall.

**The Sign of Division.**—For division the signs ÷, :, – or / are used, the first being used practically only in arithmetic. The other signs mean division only when employed in a particular way.

Thus – means division when the dividend is placed above it and the divisor below it; the other signs :, /, ÷

mean division when the dividend is placed to the left of the sign and the divisor is placed to the right.

Thus $\frac{8}{4}$ means 'divide 8 by 4.' We may write this as $8 \div 4$ or $8/4$.

$\frac{1}{2}$ means 'divide 1 by 2.'

$\frac{7}{3}$, or $7/3$, or $7 \div 3$, or $7:3$ means 'divide 7 by 3.'

$\frac{1000}{721}$, or $1000:721$, or $1000/721$ means 'divide 1000 by 721,' and so on.

So that $a:b$, $a/b$, or $\frac{a}{b}$ means '$a$ divided by $b$,' where $a$ and $b$ are any numbers whatever.

The result of a division is called a **quotient**, so that $\frac{\text{dividend}}{\text{divisor}}$ equals quotient.

The result of dividing $a$ by $b$ and multiplying the quotient by $b$ again is evidently $a$, since the second operation cancels or neutralises the first. Symbolically $\frac{a}{b}.b$ is the same as $a$.

Hence division is the inverse operation to multiplication. Thus $a \times \frac{1}{b}$ is the same as $\frac{a}{b}$.

Thus, $18 \times \frac{1}{3}$ means 18 thirds, which are 6; and $18/3$ is also 6.

$100 \times \frac{1}{10}$ means 100 tenths, that is, 10; and $100/10$ is 10.

In the same way, $a \Big/ \frac{1}{b}$ is the same as $a \times b$.

Thus, $2/\frac{1}{4}$ is 8, since there are 8 quarters in 2; and $2 \times 4$ is also 8.

$100 \div \frac{1}{10}$ is 1000, since there are 1000 tenths in 100; and $100 \times 10$ is 1000.

The statements $a \times \frac{1}{b}$ equals $a/b$

and $a \Big/ \frac{1}{b}$ equals $ab$

are true for all values of $a$ and $b$.

It should be noted that $a \div b \times c$, $a \times c \div b$, $c \times a \div b$ mean the same thing.

Thus, $16 \div 4 \times 3$ is 12. $16 \times 3 \div 4$ is 12.
$3 \times 16 \div 4$ is 12. $3 \div 4 \times 16$ is 12.

It must not be assumed that division always involves decrease. Thus $\frac{a}{b}$ may be greater than $a$.

For example, $7/\frac{1}{7}$ is the same as $7 \times 7$, that is, 49.
$12/\frac{1}{4}$ is 48.

Compare the statement about multiplication on p. 7.

**Division as Continued Subtraction.**—The original aspect of division was that of Continued Subtraction.

Ex. 1. *How many studs,* 5 *inches long, can be cut from a rod of iron* 3 *ft. long?*

Clearly we can cut 7 such studs from the rod, for $7 \times 5$ equals 35. Hence 5 inches can be subtracted from 36 inches 7 times, and leave 1 inch remaining.

Ex. 2. *How often can* $w$ *lb. of sand be removed from a heap containing* $W$ *lb.?*

We obtain the result by finding $\frac{W}{w}$ and neglecting the remainder. Thus if $W$ is 2240 and $w$ is 22, $\frac{W}{w}$ is $\frac{2240}{22}$, that is, 101 and 18 over. Therefore 101 buckets of sand weighing 22 lb. each can be removed from a heap containing 1 ton.

## EXERCISES IV.

**1.** How many times can £1. 10*s.* 0*d.* be subtracted from £100, and how much remains? This problem might occur in the following form. A man has £100 to disburse in charity, and the recipients are to have 30*s.* each. How many can he relieve, and what balance will he have for expenses?

**2.** The distance round a race track is 228 yards. How far is the winning post from the starting post in the mile race? (1 mile is 1760 yards.)

**3.** A packing case is 30 inches deep. How many layers of smaller packages 4 inches thick will it contain, and what space is left at the top?

**4.** A wall is $l$ feet long. What is its length in inches? How many bricks 9 inches long are required for the face of a stretcher course? Neglect the mortar joints.

**5.** A crucible holds 250 lb. of molten iron. How many similar castings each weighing 27 lb. can be poured from it?

**6.** How many strips of paper $2\frac{1}{2}$ inches wide can be cut from a sheet 32 inches wide? What is the width of the strip that remains?

**7.** A man's step is about 34 inches long. Assume that the length of his step does not alter, and find how many paces he takes in walking a mile.

**8.** A man has £$x$ to distribute among $n$ people, each of whom is to receive £$a$. How much will be left for expenses?

**9.** The distance between the centres of the end rivets in a row is D inches. The pitch of the rivets is $p''$. How many rivets are there altogether?

**10.** A tank holds $29w$ gallons of water. How many times can a bucket holding $2w$ gallons be filled from the tank, and how many gallons are left in the tank?

**11.** How many times can 83 be subtracted from 10000?

**12.** The distance between the centre lines of two consecutive posts in a fence is L ft., and the posts are each $x$ inches wide. There are $n$ palings each $a$ inches wide equally spaced. What is the width of each space?

**Abbreviations.**—The sign = is used to denote any of the phrases 'is the same as,' 'is equivalent to,' 'equals,' 'are equal to,' etc.

Thus, $\mathrm{I}=\frac{\mathrm{PLAN}}{33000}$ means '**I** is equal to the number obtained by dividing the product of P, L, A and N by 33000.'

The sign $\overset{\frown}{-}$ means 'is or are approximately equal to.'

$$2+\frac{1}{100000}\overset{\frown}{-}2.$$

$x+h\overset{\frown}{-}x$ if $h$ is very small compared with $x$.

The sign $\neq$ means 'is not equal to.'

Thus,
$$ab\neq a+b.$$
$$2+3(4+1)\neq(2+3)(4+1).$$

Other useful abbreviations are:

| | |
|---|---|
| $\therefore$ for 'therefore,' | $>$ for 'greater than,' |
| $\because$ for 'because,' | $\ngtr$ for 'not greater than,' |
| $+^{ve}$ for 'positive,' | $<$ for 'less than,' |
| $-^{ve}$ for 'negative,' | $\nless$ for 'not less than,' |
| *i.e.* for 'that is,' | *e.g.* for 'for example,' |
| $\angle$ for 'angle,' | $\odot$ for 'circle,' |
| $\triangle$ for 'triangle,' | $\perp^{r}$ for 'perpendicular,' |
| $\parallel$ for 'parallel,' | viz. for 'namely.' |

The meanings of these signs must be learnt and the signs used whenever possible in order to save time and space.

Thus, $4 > 2\frac{1}{5}$ means 'four is greater than two and one-fifth.

$5 \nless 2$ means 'five is not less than two.'

If $a$ is $+^{ve}$, $\therefore 2a$ is $+^{ve}$, means 'if $a$ is positive, therefore $2a$ is positive.'

## EXERCISES V.

Express each of the following as concisely as possible:

**1.** Since 5 is greater than three and three-quarters, therefore twice five is greater than seven and a half.

**2.** The angle at P is not greater than the angle at Q.

**3.** The result of dividing 10 by 5 is greater than the result of dividing 10 by 6, since 5 is less than 6.

**4.** If P is greater than Q, therefore P increased by $x$ is greater than Q increased by $x$.

**5.** Because R is less than S, R diminished by $a$ is less than S diminished by $a$.

**6.** Because $x$ equals $a$, therefore $2x$ equals $2a$ and two and one-half times $x$ is greater than twice $a$.

**7.** If $y$ is greater than $z$, then $a$ times $y$ will be greater than $a$ times $z$ as long as $a$ is positive.

**8.** The straight line joining the centre of a circle to the middle point of a chord is perpendicular to the chord.

**9.** The tangent to a circle is perpendicular to the radius through its point of contact.

**The Use of Symbols.**—It is now possible to translate simple mathematical expressions and statements into ordinary language and to do the converse, viz.: to express certain statements symbolically, *i.e.* in mathematical shorthand.

Thus, $\frac{ab}{c}+x$ means 'add $x$ to the number obtained by dividing the product of $a$ and $b$ by $c$.'

$\frac{W}{l-t}+3p$ means 'divide W by the difference of $l$ and $t$ and add three times $p$ to the quotient.'

P being greater than Q, then $\frac{P-Q}{P+Q}$ means 'divide the difference of P and Q by their sum.'

Ex. *Express in symbols:* x *divided by two, added to the sum of* y *and* z *diminished by* w *equals five and a half if* x *equals four,* y *equals three,* z *equals one and* w *equals a half.*

All this can be expressed shortly as $\frac{x}{2}+y+z-w=5\frac{1}{2}$ if $x=4$, $y=3$, $z=1$, $w=\frac{1}{2}$.

## EXERCISES VI.

**1.** Express in words $S=vt$, $P=mf$, $v-u=ft$,

$$C=\frac{E}{R+r},\quad pv=1,\quad l+\frac{m}{n}\quad\text{and}\quad\frac{x-y}{r}+\frac{r}{x-y}.$$

**2.** Express in words: 'if $p>q$, then $mp>mq$ as long as $m$ is $+^{ve}$.'

**3.** Express symbolically: (*a*) diminish $y$ by 4; (*b*) add $x$ to $y$ and divide the result by $z$; (*c*) take 4 from the sum of $3x$ and 14; (*d*) add together $x$, $x+2$ and $x+4$.

**4.** If $x$ is a whole number, what is the number next above $x$? what is the number next below $x$?

**5.** If $a$ is an even number, what are the even numbers next above and below $a$?

**6.** A man is $x$ years old. How old was he 7 years ago? How old will he be in $a$ years?

**7.** A purse contains $x$ pounds: $y$ shillings is taken from it and spent. How much remains in the purse?

**8.** The distance between two towns is $m$ miles. A cyclist leaves one town and cycles towards the other at 12 miles an hour for 2 hours. How far is he from the other town?

**9.** A girder weighs 450 lb.: from it hangs a block and tackle weighing 275 lb. What is the total load on the girder when a load of W lb. is lifted by the block?

**10.** The cross-sectional area of a rod is A square inches. If the rod supports a load of W lb., what is the load on each square inch of its cross-section.

(*Note.*—The load per unit of area, that is, on each square inch of cross-section, is called the **stress.**)

**11.** In a cylinder the steam pressure is $p$ lb. per square inch. What is the total pressure on the piston if its area is A square inches?

# CHAPTER II.

## THE DECIMAL NOTATION. THE INDEX NOTATION. THE METRIC SYSTEM. WEIGHTS AND DENSITIES.

**The Decimal Notation.**—The only digits used in calculation are 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. Any number can be expressed by a suitable use of these digits, the precise significance of any digit depending on its position.

Thus, the digits 2 and 1 are used to form the two distinct numbers 12 and 21.

12 means $1\times10+2$.

21 means $2\times10+1$.

57832 means $5\times10000+7\times1000+8\times100+3\times10+2$.

900870 means $9\times100000+8\times100+7\times10$.

The above illustrations shew that for whole numbers the right-hand digit indicates the number of units, the next to the left the number of tens, the next the number of hundreds and so on.

Reading from *left to right*, that is, taking the most important digit first and the others successively, the digit to the right of the thousands digits indicates the number of hundreds (*i.e.* of tenths of a thousand); one immediately to the right of the hundreds digit gives the number of tens (*i.e.* tenths of a hundred) and so on.

Thus the number whose digits, reading from *left to right* are $a$, $b$ is $10a+b$. If the digits, reading in the same direction, are $x$, $y$, $z$, the number is $100x+10y+z$.

**Numbers other than Whole Numbers.**—Occasions will arise for dealing with numbers less than the unit, or for numbers other than whole numbers.

Since a digit immediately to the right of the hundreds digit gives the number of tens (tenths of a hundred), and one immediately to the right of the tens digit gives the number of units (tenths of a ten), it is only logical to suppose that a digit immediately to the right of the units digit will represent the number of *tenths of a unit.*

Suppose we wish to represent 5 tens + 5 units + 5 tenths. Does 555 represent the number? Evidently not, for 555 means 5 hundreds + 5 tens + 5 units. Clearly we must adopt some method of fixing the position of the units digit. This is done by placing a dot, called a **decimal point**, immediately after the units digit. Hence the required number is written

$$55{\cdot}5.$$

The decimal point must not be confused with the dot sometimes used to denote multiplication (p. 7).

$43{\cdot}7$ means $(4 \times 10) + (3 \times 1) + 7$ tenths, and is read 'forty-three point seven.'

The digit next to the right of the tenths digit indicates hundredths (*i.e.* tenths of one-tenth).

$43{\cdot}74$ (read forty-three point seven four), means

$40 + 3 + 7$ tenths $+ 4$ hundredths.

The process can be continued, giving in succession thousandths, ten-thousandths, etc.

**Repeated Multiplication, or Division, by 10.**—In multiplying by 10, the decimal point is moved one place to the right; since multiplying by 10 changes tenths to units, units to tens, etc.

$$47{\cdot}52 \times 10 = 475{\cdot}2.$$

Since division is the inverse of multiplication, and motion to the left is the inverse of motion to the right, it is natural to suppose that in dividing by 10 the decimal point is moved one place to the left. This is evidently true, as dividing by 10 changes tens to units, units to tenths, etc.

$$47{\cdot}52 \div 10 = 4{\cdot}752.$$

Multiplying by 100 is multiplying by 10 twice over. Hence, the decimal point is moved *two* places to the right.

In dividing by 100, the decimal point is moved *two* places to the left, and so on.

$$80{\cdot}04 \times 100 = 8004.$$

$$8{\cdot}004 \div 100 = {\cdot}08004 \text{ (better written } 0{\cdot}08004\text{)}.$$

$$77{\cdot}06 \times 1000 = 77060{\cdot}0.$$

$$0{\cdot}07706 \div 100 = 0{\cdot}0007706.$$

$$879{\cdot}4 \times 10000 = 8794000{\cdot}0.$$

$$87{\cdot}94 \div 100 \times 10000 = 8794{\cdot}0.$$

Ex. *Express two and five-eighths hundredths in decimal form.*

The words expressed in figures* are

$$\frac{2\frac{5}{8}}{100} \text{ or } \frac{2{\cdot}625}{100} \text{ or } 0{\cdot}02625.$$

## EXERCISES VII.

**1.** Write in words 578·32, 57·832, 0·5783, 0·0057, 5783000.

**2.** Express in numbers: seventy thousand, eight hundred and twenty-five plus two tenths; three hundred and ninety-six plus four tenths plus five thousandths; seven plus four hundredths plus one thousandth; six hundred plus two hundredths; forty plus four thousandths; ninety plus nine hundredths plus seven thousandths; seven nine eight point four nought one.

**3.** Write down by inspection the values of: 2·002 × 1000, 17·32 ÷ 1000, 0·0045 ÷ 100, 5780 × 100000, 57·8 ÷ 100000, 3·14 × 100, 700·4 × 1000 ÷ 100, 88·88 ÷ 100000 ÷ 100.

**4.** Express the following as decimals, and find their sum:

$\frac{9}{10}$, $\frac{3}{10}$, $\frac{4}{1000}$, $\frac{7}{10000}$. M.C.U.

**5.** Write the following in decimal form and add: (*a*) three hundredths, (*b*) half of one tenth and (*c*) one and a half thousandths. M.C.U.

**6.** Write down the values of

3·56 × 10, 19·7 ÷ 10, 0·012 × 1000, 172 ÷ 1000. M.C.U.

---

* The word 'figure' sometimes refers to an illustration; sometimes to a digit of a number. The context will always shew the meaning intended.

*Express in numbers:*

**7.** The diameter of the Earth is roughly eight thousand miles.

**8.** The Sun is about ninety-three million miles from the Earth.

**9.** The wave-length of sodium light is roughly two-hundred thousandths of an inch.

**10.** A cubic centimetre is one millionth of a cubic metre and one thousandth of a litre.

**The Index Notation. Powers.**—100 is the same as 10 times 10, or $10 \times 10$; and it will be found convenient to represent 100 as $10^2$, which is read 'ten squared.'

Similarly, 1000 is the same as $10 \times 10 \times 10$, and is written $10^3$, which is read 'ten cubed.'

10000 is written $10^4$, and read 'ten to the fourth power,' or shortly, 'ten to the fourth,' and so on.

A table of these **powers** of 10 may be drawn up:

$$\begin{aligned} 1000000 &= 10^6, \\ 100000 &= 10^5, \\ 10000 &= 10^4, \\ 1000 &= 10^3, \\ 100 &= 10^2, \\ 10 &= 10^1. \end{aligned}$$

This is called the **index notation**, and the small figure in each case is the **index of the power** to which 10 is raised, or briefly, the **index**.

On examination it will be seen that each number in the left-hand column is obtained by dividing the number above it by 10, while the index of each number in the right-hand column is obtained by subtracting 1 from the index of the number next above it.

Extending the table downwards, it should be clear that the number on the left below 10 will be 1, that on the right below $10^1$ will be $10^{1-1}$, that is, $10^0$.

We may consequently say $1 = 10^0$.

Continuing the table in the same fashion, we have:

$$\tfrac{1}{10} = 10^{0-1},$$

*i.e.* $$0{\cdot}1 = 10^{-1},$$

while $$0{\cdot}01 = 10^{-1-1}, \text{ that is, } 10^{-2},$$

$$0{\cdot}001 = 10^{-3},$$

$$0{\cdot}0001 = 10^{-4},$$

and so on.

It is useful to consider the subject from a slightly different point of view.

$10000 = 10^4 = 1 \times 10 \times 10 \times 10 \times 10$, *i.e.* 1 multiplied by ten four times.

$100 = 10^2 = 1 \times 10 \times 10$, *i.e.* 1 multiplied by ten twice,

$10 = 10^1 = 1 \times 10$, *i.e.* 1 multiplied by ten once.

Hence 10 raised to a $+^{\text{ve}}$ index means that 1 is to be multiplied by ten as often as is indicated by the index. Naturally, $10^0$ means 1 not multiplied by ten at all, *i.e.* $10^0 = 1$.

Again, $-$ is the inverse of $+$, and division the inverse of multiplication. Therefore we infer that $10^{-4}$ means 1 *divided* by ten four times,

*i.e.* $$10^{-4} = \frac{1}{10 \times 10 \times 10 \times 10} = 0{\cdot}0001.$$

Similarly $10^{-2}$ means 1 divided by ten twice, *i.e.* $\dfrac{1}{10 \times 10}$ or $0{\cdot}01$.

This aspect of the index notation is clearly consistent with that first considered.

In the same way, $2^3$ means 1 multiplied by 2 three times.

$$\therefore\ 2^3 = 2 \times 2 \times 2 = 8.$$

$3^{-2}$ means 1 divided by 3 twice.

$$\therefore\ 3^{-2} = \frac{1}{3 \times 3} = \frac{1}{9}.$$

$a^0$ means 1 not multiplied by $a$ at all.

$$\therefore\ a^0 = 1.$$

**Alternative Representation of Numbers.**—Our symbols can be used in the following way:

$$\begin{aligned} 57832000 &= 5{\cdot}7832 \times 10^{7}, \\ 578{\cdot}32 &= 5{\cdot}7832 \times 10^{2}, \\ 57{\cdot}832 &= 5{\cdot}7832 \times 10^{1}, \\ 5{\cdot}7832 &= 5{\cdot}7832 \times 10^{0}, \\ 0{\cdot}57832 &= 5{\cdot}7832 \times 10^{-1}, \\ 0{\cdot}057832 &= 5{\cdot}7832 \times 10^{-2}, \\ 0{\cdot}00087832 &= 5{\cdot}7832 \times 10^{-4}, \end{aligned}$$

and so on.

In this table each number has been expressed as **the product of a number between one and ten** (viz. 5·7832) **and a power of 10.**

The index of the power of ten in any particular case should be determined by common sense. The truth of each of the following statements should be clear:

*Any number whatever can be expressed as a number between unity and ten multiplied by a power of ten.*

*In all cases the digits of the required number between one and ten are the* **same** *as the digits of the original number.*

*In the case of numbers* **greater than unity,** *the index of the power of ten is a* **positive number less by one** *than the number of digits to the* **left** *of the decimal point.*

*In the case of numbers* **less than unity,** *the index of the power of ten is* **minus** *a number* **greater by one** *than the* **number of zeros between the decimal point and the first digit other than zero of the number considered.**

The preceding rules will be found of great use when dealing with logarithms.

Thus, in 1732000 there are 7 digits to the *left* of the decimal point (which is usually omitted in whole numbers).

$$\therefore\ 1732000 = 1{\cdot}732 \times 10^{6}.$$

In 7300·1 there are 4 digits to the left of the decimal point.

$$\therefore\ 7300{\cdot}1 = 7{\cdot}3001 \times 10^{3}.$$

In 700·07 there are 3 digits to the left of the decimal point.

$$\therefore\ 700{\cdot}07 = 7{\cdot}0007 \times 10^{2}.$$

In 0·72031 there are *no* zeros between the decimal point and the first digit of the number.

$$\therefore\ 0{\cdot}72031 = 7{\cdot}2031 \times 10^{-1}.$$

In 0·007004 there are *two* zeros between the decimal point and the first digit of the number other than zero.

$$\therefore\ 0{\cdot}007004 = 7{\cdot}004 \times 10^{-3}.$$

Similarly,

$$900{\cdot}009 = 9{\cdot}00009 \times 10^{2},$$
$$0{\cdot}009 = 9{\cdot}0 \times 10^{-3},$$
$$0{\cdot}7003 = 7{\cdot}003 \times 10^{-1},$$
$$70{\cdot}03 = 7{\cdot}003 \times 10^{1},$$

and so on.

## EXERCISES VIII.

**1.** Express each of the following numbers as the product of a number between 1 and 10 by a power of 10 : 100·7, 43·17, 20·005, 473000, 999·01, 10000·01, 0·8763, 0·80072, 0·000832, 0·008007, 800·7.

**2.** Express in index form :

(*a*) The diameter of the Earth is about 8000 miles.

(*b*) Mount Everest is about 29000 feet high.

(*c*) The distance of the Sun from the Earth is about ninety-three million miles.

(*d*) The nearest fixed star is about 275000 times as far from the Earth as the Sun.

(*e*) The wave-length of sodium light is about $\frac{1}{50000}$ inch. (*Note.*—$\frac{1}{50000} = \frac{2}{100000}$.)

(*f*) Light travels at about 300000 kilometres per second.

(*g*) The velocity of a rifle-bullet is about 2000 feet per second.

(*h*) 1 cubic inch of cast-iron weighs about 0·26 lb.

(*i*) The engines of H.I.J.M. cruiser 'Kongo' develop 70000 horse-power. The ship displaces 27500 tons.

(*j*) A cubic metre is one million cubic centimetres.

(*k*) A cubic centimetre is a thousand cubic millimetres ; it is also $\frac{1}{1000}$ of a litre.

**Multiplication and Division of Powers of 10.**—Since $10^2$ means $10 \times 10$ and $10^3$ means $10 \times 10 \times 10$, it follows that:

$$10^2 \times 10^3 \text{ means } (10 \times 10) \times (10 \times 10 \times 10),$$

$$i.e.\ 10^5.$$

Similarly, $$10^2 \times 10^4 = (10 \times 10) \times (10 \times 10 \times 10 \times 10)$$
$$= 10^6,$$
$$10^7 \times 10^{11} = 10^{18},$$

and generally $$10^x \times 10^y = 10^{x+y}.$$

In the same way

$$10^4 \times 10^{-2} \text{ means } (10 \times 10 \times 10 \times 10) \div (10 \times 10),$$
$$i.e.\ 10^2, \text{ which is } 10^{4-2}.$$
$$10^5 \times 10^{-3} \text{ means } (10 \times 10 \times 10 \times 10 \times 10) \div (10 \times 10 \times 10),$$
$$i.e.\ 10^2, \text{ which is } 10^{5-3}.$$

Similarly, $$10 \times 10^{-4} \text{ means } 10 \div (10 \times 10 \times 10 \times 10),$$
$$i.e.\ \frac{1}{10 \times 10 \times 10} = 10^{-3}, \text{ which is } 10^{1-4},$$

and generally $$10^x \times 10^{-y} = 10^{x-y},$$

or $$10^x \div 10^y = 10^{x-y}.$$

Ex. 1. *Express* $20 \times 3{\cdot}5$ *as a number between* 1 *and* 10 *multiplied by a power of* 10.

$$20 = 2 \times 10;$$
$$\therefore\ 3{\cdot}5 \times 20 = 3{\cdot}5 \times 2 \times 10$$
$$= 7 \times 10.$$

Ex. 2. *Express* $620 \times 0{\cdot}05 \div 25$ *as a number between* 1 *and* 10 *multiplied by a power of* 10.

$$620 = 6{\cdot}2 \times 10^2,$$
$$0{\cdot}05 = 5 \times 10^{-2},$$
$$25 = 2{\cdot}5 \times 10.$$

$$\therefore\ 620 \times 0{\cdot}05 \div 25 = \frac{6{\cdot}2 \times 10^2 \times 5 \times 10^{-2}}{2{\cdot}5 \times 10}$$
$$= \frac{6{\cdot}2 \times 5 \times 10^0}{2{\cdot}5 \times 10}$$
$$= 6{\cdot}2 \times 2 \times 10^{0-1}$$
$$= 12{\cdot}4 \times 10^{-1}$$
$$= 1{\cdot}24 \times 10^0.$$

Ex. 3. *Find the value of* $0{\cdot}02 \times 0{\cdot}00001 \times 35 \div 0{\cdot}0004$.

$$\begin{aligned} 0{\cdot}02 &= 2 \times 10^{-2}, \\ 0{\cdot}00001 &= 1 \times 10^{-5}, \\ 35 &= 3{\cdot}5 \times 10^{1}, \\ 0{\cdot}0004 &= 4 \times 10^{-4}. \end{aligned}$$

$$\begin{aligned} \therefore \quad \frac{0{\cdot}02 \times 0{\cdot}00001 \times 35}{0{\cdot}0004} &= \frac{2 \times 10^{-2} \times 1 \times 10^{-5} \times 3{\cdot}5 \times 10^{1}}{4 \times 10^{-4}} \\ &= \frac{2 \times 1 \times 3{\cdot}5 \times 10^{-6}}{4 \times 10^{-4}} \\ &= \frac{7 \times 10^{-2}}{4} \\ &= 1{\cdot}75 \times 10^{-2} \\ &= 0{\cdot}0175. \end{aligned}$$

## EXERCISES IX.

**1.** Find the value of

(i) $10^{2} \times 10^{-4} \times 10^{5}$.

(ii) $10^{3} \times 10^{-7} \times 10^{2} \times 10^{-4}$.

(iii) $10^{4} \div 10^{7} \times 10^{8}$.

(iv) $1 \div 10^{2} \times 10^{3}$.

(v) $1 \div (10^{2} \times 10^{3})$.

**2.** Express as numbers between 1 and 10 multiplied by a power of 10

(i) $0{\cdot}013 \times 5 \times 1{\cdot}2$.

(ii) $470 \times 0{\cdot}8 \times 0{\cdot}015$.

(iii) $10{\cdot}2 \times 1{\cdot}2 \div 0{\cdot}017$.

(iv) $0{\cdot}2 \times 0{\cdot}2 \div 0{\cdot}00001$.

(v) $73{\cdot}44 \times 0{\cdot}65 \div (48 \times 25)$.

**3.** Find the value of

(i) $10000 \times 0{\cdot}001 \div (100 \times 0{\cdot}0001)$.

(ii) $0{\cdot}004 \times 20000 \div 40 \times 800$.

(iii) $36{\cdot}04 \times 1{\cdot}36 \div (0{\cdot}017 \times 800)$.

(iv) $2{\cdot}94 \times 0{\cdot}7344 \div (70 \times 0{\cdot}00126)$.

(v) $75{\cdot}2 \times 0{\cdot}048 \div (5 \times 6400)$.

**The Metric System.**—The best known system of decimal measurement is the metric system; this is invariably used in scientific work, and there is a growing tendency to use it in engineering practice.

**The standard of length** is **1 metre.**

1 metre (1 m.) = **10 decimetres** (10 dm.)
= **100 centimetres** (100 cm.)
= **1000 millimetres** (1000 mm.)

The metre is equal in length to about 39 inches.

In the metric system, the **centimetre is taken as the unit of length.**

Fig. 5 shews a portion of a rule graduated along one edge in inches and along the other in centimetres.

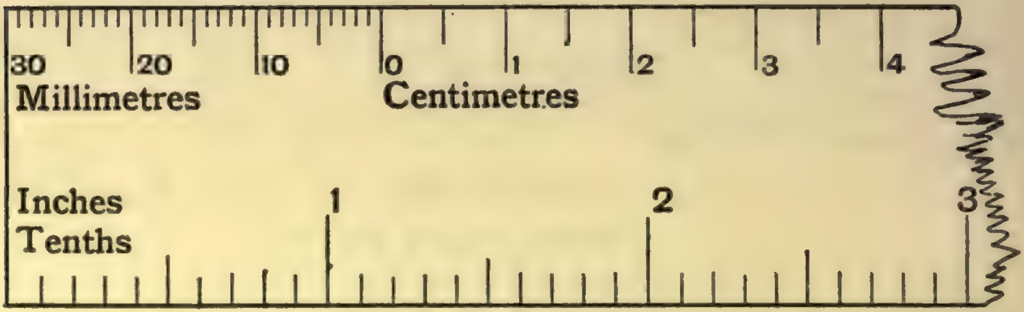


FIG. 5.

Fig. 6 shews inches divided into 8 and 16 equal parts and a decimetre divided into 10 and 100 equal parts, but not full size.

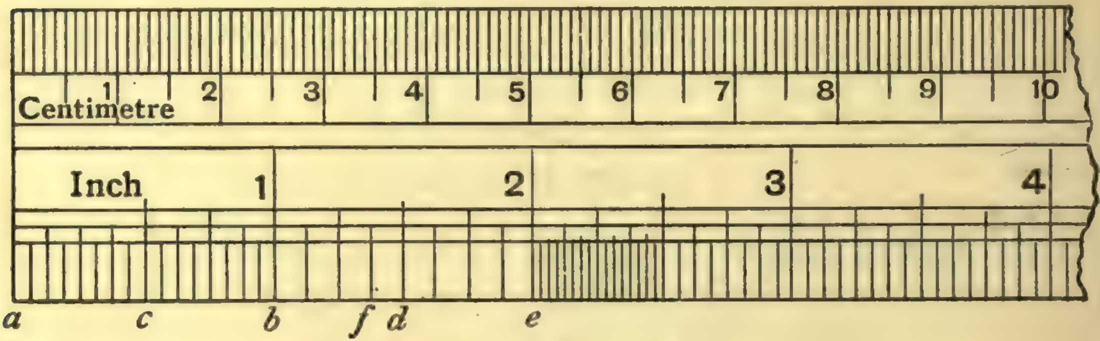


FIG. 6.

For measuring lengths much greater than a metre, a length of 1000 metres, called a **kilometre** (Km.), is used.

10 Km. $\simeq$ 6·2 miles.

The following statements should be obvious:

$$1 \text{ Km.} = 10^3 \text{ m.} = 10^5 \text{ cm.} = 10^6 \text{ mm.}$$

$$1 \text{ m.} = 10^2 \text{ cm.} = 10^3 \text{ mm.}$$

$$1 \text{ cm.} = 10 \text{ mm.}$$

$$1 \text{ mm.} = 0{\cdot}1 \text{ or } 10^{-1} \text{ cm.} = 0{\cdot}001 \text{ or } 10^{-3} \text{ m.} = 0{\cdot}000001 \text{ or } 10^{-6} \text{ Km.}$$

$$1 \text{ cm.} = 0{\cdot}01 \text{ or } 10^{-2} \text{ m.} = 0{\cdot}00001 \text{ or } 10^{-5} \text{ Km.}$$

$$1 \text{ m.} = 0{\cdot}001 \text{ or } 10^{-3} \text{ Km.}$$

**A millionth of a metre** is sometimes used for measuring very small lengths, and **is called a micron.**

$$1 \text{ micron} = 10^{-6} \text{ m.}$$

**The unit of weight is 1 gram** (gm.). As in the case of lengths, there are multiples and sub-multiples of this unit.

**1 kilogram** (Kg.) $= 10^3$ grams (gm.) $=$ **$10^6$ milligrams** (mg.).

$$1 \text{ gm.} = 10^3 \text{ mg.}$$

$$1 \text{ mg.} = 0{\cdot}001 \text{ or } 10^{-3} \text{ gm.} = 0{\cdot}000001 \text{ or } 10^{-6} \text{ Kg.}$$

$$1 \text{ gm.} = 0{\cdot}001 \text{ or } 10^{-3} \text{ Kg.}$$

$1 \text{ Kg.} \bumpeq 2{\cdot}2 \text{ lb.}$

**The unit of area is 1 square centimetre** (sq. cm.), and is the amount of surface covered by a square whose side is 1 cm.

$$1 \text{ Hectare} = 10^4 \text{ sq. m.} = 10^8 \text{ sq. cm.}$$

$$1 \text{ Are} = 10^2 \text{ sq. m.} = 10^6 \text{ sq. cm.}$$

$$1 \text{ sq. mm.} = 0{\cdot}01 \text{ or } 10^{-2} \text{ sq. cm.}$$

$$1 \text{ Hectare} \bumpeq 2{\cdot}5 \text{ acres.}$$

**The unit of volume is 1 cubic centimetre** (c.c.), and is the space occupied by a cube whose side is 1 cm.

**1 litre** $= 10^3$ c.c.

$$1 \text{ c.c.} = 0{\cdot}001 \text{ or } 10^{-3} \text{ litres.}$$

$$10 \text{ litres} \bumpeq 8{\cdot}8 \text{ quarts.}$$

**Summary.**—The preceding statements regarding length and weight are summarised in the following table:

| Standard multiplied by | Length. | Weight. |
|---|---|---|
| $10^6$ | — | (Tonne) |
| $10^3$ | Km. | Kg. |
| $10^2$ | (Hectometre) | (Hectogram) |
| $10^1$ | (Decametre) | (Decagram) |
| $10^0$ | m. | gm. |
| $10^{-1}$ | (Decimetre) | (Decigram) |
| $10^{-2}$ | cm. | (Centigram) |
| $10^{-3}$ | mm. | mg. |
| $10^{-6}$ | (Micron) | — |

The terms in brackets are seldom used.

Ex. 1. *Express in Km., mm., each of the following:* (*a*) 4 *m.* 2 *cm.;* (*b*) 13 *Km.* 54 *m.* 10·7 *cm.*

(*a*) 4 m. 2 cm. = 4·02 cm. = 0·00402 Km.,

4 m. 2 cm. = 402 cm. = 4020 mm.

(*b*) 13 Km. 54 m. 10·7 cm. = 13 Km. 54·107 cm.

= 13·054107 Km.,

13 Km. 54 m. 10·7 cm. = 13054 m. 10·7 cm.

= 13054107 mm.

Ex. 2. *Find the weight of a litre of water in kilograms, if* 1 *c.c. of water weighs* 1 *gm.*

1 litre of water $=10^3$ c.c., which weighs $10^3$ gms. or 1 Kg.;

$\therefore$ 1 litre of water weighs 1 Kg.

Ex. 3. *How many litres are there in a cubic metre?*

1 metre $=10^2$ cm.;

$\therefore$ 1 cubic metre $=(10^2)^3$ c.c. $=10^6$ c.c.

1 litre $=10^3$ c.c.;

$\therefore$ 1 cubic metre $=10^6/10^3$ litres $=1000$ litres.

## EXERCISES X.

1. Express in the shortest form as (*a*) metres, (*b*) kilometres, (*c*) centimetres, each of the following :

5 m. 4 cm. 2 mm., 2 Km. 4 m., 4 m. 5 dm. 4 mm.

2. Add together 4 m. 3 cm., 7 dm. 6 cm. 1 mm. and 4 m. 2 cm. 8 mm.

3. Reduce 12·45 m. to cm.

4. What distance is covered by 100 rods each 4·5 cm. long placed end to end?

5. Express in the shortest form as (*a*) milligrams, (*b*) kilograms, (*c*) grams, each of the following :

5 gm. 85 mg., 24 Kg. 84 gm., 7 Kg. 45 gm. 24 mg.

6. Add together 24 Kg. 4 gm., 5 Kg. 72 gm. 8 mg. and 17 Kg. 84 gm. 22 mg.

7. Simplify 8 m. 4 cm. 5 mm. + 22 Km. 80 m. − 18 Km. 5 m. 15 cm. − 84 m. 5 mm., and express the result in metres.

8. How many square cm. are there in 5 sq. m.?

9. Complete the following table :

1 sq. metre = sq. cm. = sq. mm.,
1 sq. cm. = sq. mm.

10. How many cubic cm. are there in a cubic m.? How many in a cubic Km.?

11. How many times can a litre flask be filled from a tank holding $10^{12}$ c.c.?

12. How many mg. are there in 1 Kg.?

13. What multiplier should be used to convert
(*a*) cm. to Km., (*b*) m. to mm., (*c*) gm. to Kg.,
(*d*) gm. to mg., (*e*) Are to square cm.?

**Weights and Densities.**—Any volume of a given substance has a certain weight. It is convenient to use the term **density** to denote the weight of unit volume; *i.e.* **the density of a substance is the weight in grams of 1 cubic centimetre of that substance.**

Sometimes densities are given in pounds per cubic foot, or in pounds per cubic inch.

DENSITIES OF TYPICAL SUBSTANCES.

| Substance. | Density gm. per c.c. | Density lb. per c. ft. |
|---|---|---|
| Water - - | 1 | 62·3 |
| Mercury - - | 13·6 | 850 |
| Copper - - | 8·9 | 550 |
| Lead - - | 11·4 | 710 |
| Cast Iron - - | 7·2 | 450 |
| Wrought Iron - | 7·6 | 480 |
| Brass - - | 8·2 | 520 |
| Aluminium - | 2·7 | 168 |
| Milk - - - | 1·03 | 64·2 |
| Paraffin Oil - | 0·93 | 58 |
| Granite - - | 2·8 | 170 |
| Cement - - | 3·0 | 180 |

These numbers are approximate only.

**The specific gravity** (S.G.) of a body is its weight divided by the weight of an equal volume of water, the weights being expressed in the same units. Since 1 c.c. of water weighs 1 gram, the number expressing the specific gravity of a body is the same as the number of grams in 1 c.c. of the body.

The density of any substance is determined experimentally by finding the weight and volume of any specimen, and calculating the weight of unit volume. The weight of the specimen is found by using scales, or a spring balance, and the volume is determined most conveniently by seeing how much water the specimen displaces. Details of such experiments will be found in any book on Practical Physics.

EX. 1. *The weight of a body is* w *gm. and its volume* v *c.c. What is its specific gravity? What is its density?*

The weight of v c.c. of water = v grams.

$$\therefore \text{ S.G. (specific gravity)} = \frac{w}{v}.$$

The density is the weight of 1 c.c. of the body,

*i.e.* $\frac{w}{v}$ gm. per c.c.

Ex. 2. *A body occupying* $v$ *cubic ft. weighs* $w$ *tons. Find its density and specific gravity.*

$w$ tons $= 2240w$ lb.

$$\therefore \text{ the density of the body} = \frac{2204w}{v} \text{ lb. per cubic foot.}$$

The weight of $v$ cubic ft. of water $= 62{\cdot}3 \times v$ lb.

$$\therefore \text{ S.G. of body} = \frac{2420w}{62{\cdot}3v}.$$

## EXERCISES XI.

**1.** What is the volume of 23 gm. of water?

**2.** What is the weight of 17·6 c.c. of water? What is the weight of a litre of water?

**3.** A bottle, whose weight is 13·65 gm. when empty, weighs 63·65 gm. when filled with water. What is the capacity of the bottle in cubic cm.?

**4.** The bottle in the last question weighs 694·8 gm. when filled with mercury. Calculate the density of mercury?

**5.** A litre bottle is filled with paraffin. How many gm. of paraffin does it hold?

**6.** What is the weight of 100 c.c. of mercury?

**7.** A body weighs $W$ gm. and its volume is $v$ c.c. What is the density of the body?

**8.** The density of a body is $\rho$ gm. per c.c. and its volume is $V$ c.c. What is its weight? ($\rho$ is the Greek letter 'ro.')

**9.** The S.G. of a body is $s$ and its volume is $v$ c.c. What is its weight?

**10.** If 28 cubic inches of water weigh 1 lb. and the specific gravity of a body is 4, how much do 20 cubic inches of the body weigh?

**11.** A vessel holds a cubic foot of water. What weight of mercury will it hold? What weight of milk will it hold?

**12.** Use the given value of the density of mercury to find the pressure at the base of a column of mercury 760 mm. high and 1 sq. cm. in area.

# CHAPTER III.

## SIGNIFICANT FIGURES. ROUGH CHECK. CONTRACTED METHODS OF CALCULATION.

**Significant Figures.**—Consider the following expressions for the same weight:

(*a*) 1·57 Kg.

(*b*) 1570 gm.

(*c*) 1570000 mg.

To express each weight, the same digits, 1, 5, 7, are used; the zeros or noughts and the decimal point merely shewing the size of the units employed. Thus, we may say,

(*a*) is 157 hundredths of a kilogram.

(*b*) is 157 ten-grammes.

(*c*) is 157 ten-thousand milligrams.

The digits 1, 5, 7 are called the **significant figures.** Each of the above weights is said to be correct to three significant figures.

A weight of 653 lb. is said to be 650 lb., correct to two significant figures.

A length of 0·6264 metre would be given

as 0·626 m., correct to three significant figures;

as 0·63 m., correct to two significant figures;

as 0·6 m., correct to one significant figure.

**The rule is, increase the last figure retained by 1 when the next figure is 5 or more than 5, but leave the last figure retained unaltered when the next figure is less than 5.**

Ex. 1. *The distance between two points is given as* 13·709 *in., and it may be as much as* 0·002 *in. wrong. To how many significant figures is the measurement correct?*

The actual distance may be as much as 13·709 + 0·002, *i.e.* 13·711 in., or as little as 13·709 − 0·002, *i.e.* 13·707 in.

In each case 13·709, 13·711, 13·707 are written as 13·71 **correct to four significant figures.** Hence the given measurement is correct to four significant figures.

Ex. 2. *The amount of sunshine on a given day was given as* 4·8 *hr.; assuming this to be correct to two significant figures, find the greatest and least amount of sunshine possible on that day.*

Any number less than 4·85 and greater than 4·75, *e.g.* 4·849, would be written as 4·8 correct to two significant figures.

Hence the greatest amount of sunshine possible ≏ 4·85 hr.
The least amount of sunshine possible = 4·75 hr.

The method of using significant figures may be better appreciated by considering that although there is no numerical difference between, say, 6 ft. and 6·00 ft., the first might be used as a rough approximation to the average height of a number of men, while the second might be used to express the fact that a certain height had been measured with such accuracy that we could confidently say it did not differ from 6 feet by more than five thousandths of a foot.

## EXERCISES XII.

**1.** How many significant figures are used in these statements?

(*a*) The diameter of the earth is 8000 miles.
(*b*) The velocity of sound in air at 20° C. is 344 metres per second.
(*c*) One litre of dry air weighs 1·293 gm.
(*d*) The coefficient of linear expansion of steel ≏ 0·000011.
(*e*) 1 cm. ≏ 0·3937 in.
(*f*) 1 in. ≏ 2·54 cm.
(*g*) 1 lb. ≏ 453·4 gm.
(*h*) 1 Kg. ≏ 2·205 lb.
(*k*) The density of wrought iron is 7·8 gm. per c.c.
(*l*) The density of aluminium is 2·6 gm. per c.c.

**2.** Express each number in Exercise 1 as a number between 1 and 10 multiplied by a power of 10.

**3.** Express 587·1, 93·678, 20·21, 20·05, 0·008723, 876650, 1·00047, 4444·4, 400·7, 0·9197 :

(*a*) correct to 3 significant figures,
(*b*) correct to 2 significant figures,
(*c*) correct to 1 significant figure.

**4.** Write down 6728·6 metres, correct (*a*) to the nearest metre, (*b*) to three significant figures, (*c*) to the nearest kilometre.

**5.** In 1901 the total area of England was given as 32,578,178 acres ; the population as 30,811,420. Write down the acreage and population, correct to (*a*) 4 significant figures, (*b*) correct to the nearest thousand, (*c*) correct to the nearest hundred thousand.

**6.** In 1903 the gross rental from land in England was estimated at £224,845,756. Write this rental, (*a*) correct to 4 significant figures, (*b*) correct to the nearest thousand, (*c*) correct to the nearest 100000.

In general, it is unnecessary to give more than *three* or at most *four* significant figures in order to express the result of any measurement, whether it be of length, weight, speed, time, etc. It is only in the most accurate scientific work that more than four figures are required.

It may be taken for granted that the calculations in this book—unless otherwise specified—are correct to **not more than four significant figures.**

**Contracted Calculation.**—It is now to be considered what modifications, if any, are necessary in the ordinary methods of working examples in the first four rules of arithmetic, so that useless labour may be avoided.

In the first place, **a rough calculation** should be made in order to reduce to a minimum the possibility of making an error in the position of the decimal point, and of giving results which are ridiculous.

Ex. 1. *Calculate roughly* $187{\cdot}3+72{\cdot}94+0{\cdot}00876+3{\cdot}127$.

The sum is approximately $190+70$, *i.e.* 260, the last two numbers being negligible in comparison with the first two.

Ex. 2. *Calculate roughly* $789{\cdot}62-5{\cdot}873$.

$$789{\cdot}62 \mathrel{\overset{\frown}{-}} 790 \text{ and } 5{\cdot}873 \mathrel{\overset{\frown}{-}} 6.$$

$\therefore$ the result $\mathrel{\overset{\frown}{-}} 790-6$ or 784,

*i.e.* 780 to two significant figures.

Ex. 3. *Calculate roughly* $23{\cdot}59 \times 6{\cdot}007 \times 0{\cdot}0008243$.

$$\begin{aligned}
23{\cdot}59 &\bumpeq 24\text{ ; } \textit{i.e. } 2{\cdot}4 \times 10 \text{ (see p. 22)},\\
6{\cdot}007 &\bumpeq 6,\\
0{\cdot}0008243 &\bumpeq 0{\cdot}0008\text{ ; } \textit{i.e. } 8 \times 10^{-4}.\\
\therefore \text{ product} &\bumpeq 2{\cdot}4 \times 10 \times 6 \times 8 \times 10^{-4}\\
&\bumpeq 2{\cdot}4 \times 6 \times 8 \times 10^{1-4}\\
&\bumpeq 14 \times 8 \times 10^{-3}\\
&\bumpeq 110 \times 10^{-3}\\
&\bumpeq 0{\cdot}11.
\end{aligned}$$

Ex 4. *Calculate roughly* $87{\cdot}3 \div (3{\cdot}12 \times 0{\cdot}00876)$.

Reference to p. 82 will shew that we have to divide $87{\cdot}3$ by the product of $3{\cdot}12$ and $0{\cdot}00876$.

$$\begin{aligned}
87{\cdot}3 &\bumpeq 90 = 9 \times 10\\
3{\cdot}12 &\bumpeq 3\\
0{\cdot}00876 &\bumpeq 0{\cdot}009 = 9 \times 10^{-3}.\\
\therefore \text{ result} &\bumpeq \frac{9 \times 10}{3 \times 9 \times 10^{-3}}\\
&\bumpeq \frac{9}{3 \times 9} \times 10^{1+3}\\
&\bumpeq \tfrac{1}{3} \times 10^{4}\\
&\bumpeq 0{\cdot}3 \times 10^{4}\\
&\bumpeq 3 \times 10^{3}\\
&\bumpeq 3000.
\end{aligned}$$

**Addition.**—In adding a column of numbers, the decimal points must be in a vertical line, so that tens are in one column, units in another, tenths in a third, and so on. In adding numbers in a row, care must be taken to add together corresponding digits.

Ex. 1. *Find, to four significant figures, the sum of* 187·3, 72·94, 0·00876 *and* 3·127.

Example 1 (p. 34) shews that the result is approximately 260. Since four significant figures are required, the answer must shew one decimal place. But as something may be 'carried' from the addition of the hundredths digit, two decimal places are retained in the working.

$$\begin{array}{r} 187{\cdot}3\phantom{0} \\ 72{\cdot}94 \\ 0{\cdot}01 \\ 3{\cdot}13 \\ \hline 263{\cdot}38 \end{array}$$

**Result,** 263·4 to four significant figures.

In working, each number is written down, correct to two decimal places.

Then say, performing the additions mentally and emphasising the figures in thick type,

3, 4, **8**; write **8**.
1, 10, **13**; write **3** and carry **1**.
1, 4, 6, **13**; write **3** and carry **1**.
1, 8, **16**; write **6** and carry **1**.
1, **2**; write **2**.

Ex. 2. *Find* 7·162 + 748·5 + 26·86 + 9·005 *to four significant figures.*

**Result,** $\bumpeq 750 + 30$; *i.e.* 780.

The answer requires 1 decimal place.

$$\begin{array}{r} 7{\cdot}16 \\ 748{\cdot}5\phantom{0} \\ 26{\cdot}86 \\ 9{\cdot}01 \\ \hline 791{\cdot}53 \end{array}$$

**Result,** 791·5 to four significant figures.

Ex. 3. *Find, to the nearest shilling, the sum of* £117. 18*s.* 4*d.*, £2. 4*s.* 6½*d.*, 17*s.* 8*d. and* 8*s.* 9½*d.*

**Result,** $\bumpeq$ £120.

$$\begin{array}{rcrcr} \pounds. & & s. & & d. \\ 117 & . & 18 & . & 4 \\ 2 & . & 4 & . & 7 \\ & & 17 & . & 8 \\ & & 8 & . & 10 \\ \hline 121 & . & 9 & . & 5 \end{array}$$

The answer is £121. 9*s.* 0*d.* to the nearest 1*s.*

Ex. 4. *Add together* 7·3, 2, 0·2509, 0·0678, *and give the result to three significant figures.*

**Result**, ≏ 7 + 2 ; *i.e.* 9.

7·3 + 2 + 0·251 + 0·068 = 9·619.

Remembering that hundredths must be added together, tenths to tenths, and so on, say,

8, **9** ; write down **9**,
6, **11** ; „ „ **1** and carry **1**,
1, 1, 3, **6** ; „ „ **6**,
2, **9** ; „ „ **9**.

The **result**, correct to three significant figures, is 9·62.

**Rule for Contracted Addition.—Retain one more decimal place in the working than the number of decimal places required in the result.** Each sum should be preceded by a rough calculation.

## EXERCISES XIII.

Calculate the following to four significant figures :

**1.** 400·5 + 180700 + 23·06 + 0·0289 + 1·234.

**2.** 36·79 + 2·853 + 20·01 + 0·01976.

**3.** 837100 + 37·18 + 810·7 + 0·8392 + 247·8.

**4.** 5·2 + 630 + 19000 + 0·5 + 8·23 + 0·012.

**5.** 811·7 + 71·18 + 0·00897 + 3·714.

**6.** 0·000839 + 0·001732 + 0·00008837 + 0·0889.

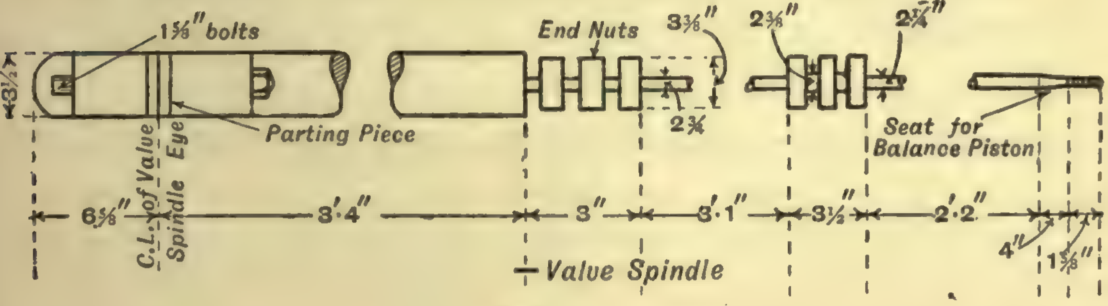


FIG. 7.

**7.** The dimensioned plan of a valve spindle is given in Fig. 7. Find the overall distance from the centre of the eye to the end of the valve rod.

**8.** Fig. 8 is one view of a reversing shaft for a triple expansion engine. Find

(i) The distance between the centre lines oi the extreme swinging links.

(ii) The overall length of the shaft.

(iii) The distance between the centre lines of the radius arms.

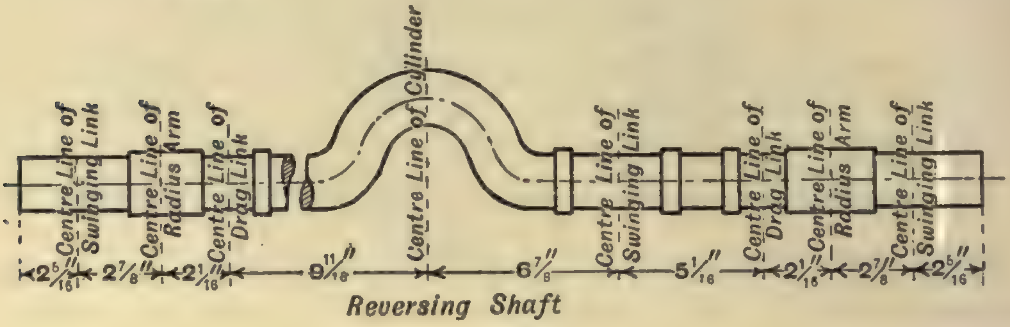


FIG. 8.

**9.** Fig. 9 gives the elevation of a Wyper shaft for the same engine. Find

(i) Its overall length.

(ii) The distance between the centre lines of the Wypers.

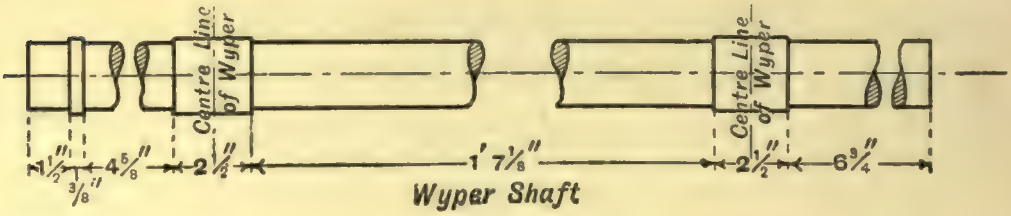


FIG. 9.

**Subtraction.**—The rules given for addition apply here. It is advisable to use the **Complementary** or **Shop** method for subtraction, as illustrated in the following examples.

Ex. 1. *From* 398·7 *take* 28·6.

Rough calculation, $398{\cdot}7 - 28{\cdot}6 \doteqdot 400 - 30 \doteqdot 370$.

$$\begin{array}{r} 398{\cdot}7 \\ 28{\cdot}6 \\ \hline 370{\cdot}1 \end{array}$$

**Result** is 370·1.

Say, 6 and **1** make 7,
8 „ **0** „ 8,
2 „ **7** „ 9,
0 „ **3** „ 3,

emphasising and writing down the figures printed in thick type.

Ex. 2. *From* £8. 14*s.* 7*d.* *take* £3. 17*s.* 9*d.*

Rough calculation, £8. 14*s.* 7*d.* – £3. 17*s.* 9*d.* ≏ £9 – £4 ≏ £5.

£8. 14*s.* 7*d.* – £3. 17*s.* 9*d.* = £4. 16*s.* 10*d.*

Say, 9*d.* and **10d.** make 1*s.* 7*d.*; write 10*d.* and carry 1*s.*
1*s.*, 18*s.* and **16s.** make £1. 14*s.* 0*d.*; write 16*s.* and carry £1.
£1, £4 and **£4** make £8; write £4.

Ex. 3. *Subtract* 8·234 *from* 121·7, *giving the result to four significant figures.*

Rough calculation, 121·7 – 8·234 ≏ 120 – 8 ≏ 110.

The result will have one decimal place; *two* decimal places are therefore retained in the working.

$$\begin{array}{r}121{\cdot}7\\ 8{\cdot}23\\ \hline 113{\cdot}47\end{array}$$

**Result** is 113·5 to four significant figures.

Say, 3 and **7** make 10; write 7 and carry 1.
1, 3 and **4** make 7; write 4.
8 and **3** make 11; write 3 and carry 1.
1 and **1** make 2; write 1.
0 and **1** make 1; write 1.

## EXERCISES XIV.

Using the complementary method of subtraction, find the results of the following:

**1.** 2003 – 1987. **2.** 8·430 – 2·019.

**3.** £139. 17*s.* 9½*d.* – £40. 19*s.* 11*d.*

**4.** £28. 18*s.* 6*d.* – £9. 7*s.* 7½*d.*

**5.** 8·467 – 0·3025, correct to four significant figures.

**6.** 91·23 – 81·47, correct to the nearest unit.

**7.** 139·85 – 130·21, correct to two significant figures.

**8.** 2 Km. 3 m. 16 cm. – 4·74 m., correct to the nearest metre.

**9.** 69·812 gm. – 17·577 gm., (*a*) correct to the nearest 10 milligrams, (*b*) correct to the nearest tenth of a kilogram.

**10.** 1000 – 225 – 154 – 53 – 119.

**11.** 987·5 – 89·8 – 123·2 – 279·8, correct to three significant figures.

**12.** 9172·37 – 8391·7 + 9·987 – 3·434, correct to four significant figures.

**Generalised Addition and Subtraction.**—The rule that like quantities alone may be added or subtracted, and consideration of the methods of adding and subtracting sums of money, etc., should make it easy to understand that the sum of

$7a+4b+c$, $8a+2b+4c$ and $a+b+c$ is $16a+7b+6c$.

As positive and negative quantities are essentially opposite in kind, the sum of $7x$, $x$, $-3x$, $2x$, $-8x$ and $-x$ is the same as the sum of $10x$ and $-12x$, and a reference to p. 4 will shew that this sum may be written as $-2x$.

Ex. 1. *Add together*

$$a+b+c,\quad 5a-3b+2c \quad\text{and}\quad -2a+b-c.$$

$$\begin{array}{r}
a+\phantom{3}b+\phantom{2}c \\
5a-3b+2c \\
-2a+\phantom{3}b-\phantom{2}c \\
\hline
4a-\phantom{3}b+2c
\end{array}$$

**Process.**—Write like terms in columns. Then say,

$$-2a,\ 3a,\ 4a\ ;\quad b,\ -2b,\ -b\ ;\quad -c,\ +c,\ +2c\ ;$$

mentally performing the addition or subtraction, as indicated by the + or − sign. Thus, in adding the '$a$' column, $-2a+5a=3a$, since the result is that obtained by **taking 2a from 5a**; and **3a**$+$**a**$=$**4a**. Similarly with the '$b$' column, $+b-3b=-2b$, since the result is that obtained by **taking 3b from b**; and $-$**2b**$+$**b**$=-$**b** in the same way.

Ex. 2. *Subtract* $3x+y-2z$ *from* $4x-y+z$.

$$\begin{array}{r}
4x-\phantom{2}y+\phantom{3}z \\
3x+\phantom{2}y-2z \\
\hline
x-2y+3z
\end{array}$$

Say, $3x$ and **x** make $4x$ ; write x.
$y$ and $-$**2y** make $-y$ ; write $-2y$.
$-2z$ and **3z** make $+z$ ; write $+3z$.

## EXERCISES XV.

Find the sum of

1. $2p-2q+3r$, $p+q-r$.
2. $2x-3y+4z$, $6x-3y+z$, $x+y+z$.
3. $x-y+z$, $2x+3y-z$, $x-2y+z$.

4. $2{\cdot}5a-3{\cdot}2b$, $3{\cdot}5a+2{\cdot}3b$.
5. $a+b+c$, $2{\cdot}2a+3{\cdot}1b-4{\cdot}5c$, $2{\cdot}5a+b-3{\cdot}1c$.

Perform the following subtractions:

6. From $6a+2b$ take $4a+b$.
7. From $8y+4z$ take $6y+5z$.
8. From $3x-y+3z$ take $4x+y-z$.
9. From $2{\cdot}5a+3b-2{\cdot}7c$ take $1{\cdot}9a-4{\cdot}5b+4c$.

Find the value of

10. $2a-3b+c+(a-b-c)-(2a+3b-c)$ when
    (i) $a=1$, $b=2$, $c=1$.
    (ii) $a=0$, $b=-2$, $c=1$,
    (iii) $a=-10$, $b=5$, $c=-5$.

**Multiplication.**—First make a rough calculation, so that the *magnitude* of the result may be gauged. The actual calculation should be performed in the way illustrated in the following examples, the decimal point being neglected at first, and afterwards inserted in its correct position before, among, or after, the significant figures of the result, in accordance with the rough calculation already made.

Ex. 1. *Multiply* $25{\cdot}3$ *by* $0{\cdot}217$, *correct to three significant figures.*

Rough calculation, $25{\cdot}3\times 0{\cdot}217 \bumpeq 25\times 0{\cdot}2$ or $5{\cdot}0$.

$$\begin{array}{ccccc}
2 & 5 & \cancel{3} & & \\
 & & 2 & 1 & 7 \\
\hline
5 & 0 & 6 & & \\
 & 2 & 5 & 3 & \\
 & 1 & 7 & 7 & \\
\hline
5 & 4 & 9 & 0 &
\end{array}$$

**Result,** $5{\cdot}49$.

**Explanation.**—The first significant figure of the multiplier is placed below the right-hand figure of the multiplicand. As three figures are required in the result, four are retained in the working.

The top line is first multiplied by the left-hand digit of the multiplier, this being the most important digit.

Say, $2\times 3 =$ **6**; write **6** below the 2 of multiplier.
$2\times 5 =$ **10**; write **0** next to the left, carry 1.
$2\times 2+1=$ **5**; write **5** next.

Now say, 1 × 3 = **3** ; write **3** below the 1 of multiplier.
1 × 5 = **5** ; write **5** next.
1 × 2 = **2** ; write **2** next.

Note there are now four figures in the result,

*i.e.* 253 × 21 = 5060 + 253 = 5313.

Next **cross out the right-hand digit 3** of multiplicand, and say :

7 × 3 = 21 ; do not write anything, but carry 2.
7 × 5 + 2 = **37** ; write **7** under the 3 of last line, carry 3.
7 × 2 + 3 = **17** ; write **17**.

Adding, the figures of the result are 5490. The rough calculation gave 5·0. Therefore the **result,** correct to 3 significant figures, is 5·49.

Ex. 2. *Evaluate* 25·376 × 0·210814, *correct to four significant figures.*

Rough calculation, 25·376 × 0·210814 ≏ 25 × 0·2 or 5·0 ; 5 figures are retained in the working.

$$\begin{array}{r}
2\cancel{5}\cancel{3}\cancel{7}\cancel{6}\phantom{0000} \\
21081 \\
\hline
50752\phantom{0000} \\
2538\phantom{0000} \\
202\phantom{0000} \\
3\phantom{0000} \\
\hline
53495\phantom{0000}
\end{array}$$

**Result** is 5·350.

**Explanation.**—The first significant figure 2 of the multiplier is placed below the right-hand digit 6 of top line.

Say, 2 × 6 = **12** ; write **2**, carry 1.
2 × 7 + 1 = **15** ; write **5**, carry 1.
2 × 3 + 1 = **7** ; write **7**.
2 × 5 = **10** ; write **0**, carry 1.
2 × 2 + 1 = **5** ; write **5**.

**Cross out the right-hand digit 6** in the top line.

Say, 1 × 6 = 6 ; do not write anything, but since 6 is nearer 10 than 0, carry **1**.

Say, 1 × 7 + 1 = **8** ; write **8** below last digit of line above.
1 × 3 = **3** ; write **3** next, and so on.

**Now cross out the 7** in the top line and multiply by 0. This gives nothing.

**Now cross out the 3** in the top line and multiply by 8.

Say, $8 \times 3 = 24$. This is 20 to one significant figure. Do not write anything, but carry 2.

$8 \times 5 + 2 = \mathbf{42}$; write **2**, carry 4.

$8 \times 2 + 4 = \mathbf{20}$; write **20**.

**Now cross out the 5** in the top line and multiply by 1.

Say, $1 \times 5 = 5$; therefore carry 1.

$1 \times 2 + 1 = \mathbf{3}$; write **3**.

Adding, the figures of the result are 53495.

The rough calculation gives 5·0.

∴ the **result** is 5·350, correct to four significant figures.

Ex. 3. *Evaluate* 47·6952 × 27·132 *to three significant figures.*

$$47{\cdot}6952 \times 27{\cdot}132 \backsimeq 48 \times 30 \backsimeq 1400.$$

$$\begin{array}{r} 4\cancel{7}\cancel{7}\cancel{0}\phantom{00} \\ 2713 \\ \hline 9540\phantom{00} \\ 3339\phantom{00} \\ 48\phantom{00} \\ 14\phantom{00} \\ \hline 12941\phantom{00} \end{array}$$

**Result,** 1290.

## EXERCISES XVI.

Find the product of

1. 2·37 and 3·16, correct to three significant figures.
2. 67·35 and 0·3472, „ „ „
3. 6·735 and 34·72, „ „ „
4. 415·7 and 2·078, „ „ „
5. 844·15 and 2·075, „ the nearest unit.
6. 39·37 and 43560, „ „ thousand.
7. 8762·8 and 13089, „ „ million.

Calculate to four significant figures :

8. $3978000 \times 0{\cdot}008736$.
9. $47{\cdot}895 \times 89{\cdot}574$.
10. $51{\cdot}372 \times 0{\cdot}0004508$.
11. $0{\cdot}008966 \times 0{\cdot}08898$.
12. $3{\cdot}1416 \times 8{\cdot}994$.

Answer the following:

**13.** Two men measure a room. One says it is 17·3 ft. long and 12·7 ft. wide; the other, that it is 17·4 ft. long and 12·6 ft. wide. What would each man probably give as the area? What would it be reasonable to take as the area?

**14.** Find the value of $ft$ when $f = 32{\cdot}18$ and $t = 18{\cdot}9$, correct to three significant figures.

**15.** Evaluate $Wh$ when $W = 2567$ and $h = 1758$, correct to three significant figures.

**Division.**—First make a rough calculation, so that the magnitude of the quotient may be gauged. The actual division should be performed in the way illustrated in the following examples, the decimal point being neglected at first, and afterwards inserted in its correct position in the quotient, in accordance with the rough calculation already made.

Ex. 1. *Divide* 26·216 *by* 113·8, *correct to three significant figures.*

Rough calculation, $\dfrac{26{\cdot}216}{113{\cdot}8} \backsimeq \dfrac{26}{110} \backsimeq 0{\cdot}2.$

$$\begin{array}{r|l} \multicolumn{1}{r}{} & \multicolumn{1}{l}{\phantom{0}2304} \\ \cline{2-2} 1\cancel{1}\cancel{3}\cancel{8} & 2622 \\ & \phantom{0}346 \\ & \phantom{000}5 \end{array} \qquad \textbf{Result, } 0{\cdot}230.$$

**Explanation.**—Four figures are kept in the working. Draw a vertical line and place the divisor on the left and the dividend on the right. Draw a horizontal line above the dividend.

The first figure of the quotient is 2.

Write 2 above the last digit of the contracted dividend.

Say, $2 \times 8 = 16$ and **6** are 22; write **6** and carry 2.

$2 \times 3 + 2 = 8$ and **4** are 12; write **4** and carry 1.

$2 \times 1 + 1 = 3$ and **3** are 6; write **3**.

$2 \times 1 = 2$ and **0** are 2; there is nothing to write.

Now **cross out the right-hand digit, 8**, of the divisor, and divide 346 by 113.

The next figure of the quotient is 3; write this down next to the 2, and say:

$3\times 8 \quad = 24$; do not write anything, but carry 2.
$3\times 3+2=11$ and **5** are 16; write **5** and carry 1.
$3\times 1+1=4$ and **0** are 4.
$3\times 1 \quad =3$ and **0** are 3; there is nothing to write.

**Strike out the 3** of the divisor and divide 5 by 11.

This will not go. So write **0** as next figure of quotient.

**Strike out the 1** of the divisor and divide 5 by 1.

The result is **not** 5 but 4, as $1\cancel{1}\times 5 \bumpeq 6$. The next figure of quotient is therefore 4.

The rough calculation shews that the result is approximately 0·2. Hence the required **result** is 0·230.

Ex. 2. *Divide* 25·376 *by* 0·210814, *correct to four significant figures.*

Rough calculation, $\dfrac{25{\cdot}376}{0{\cdot}210814} \bumpeq \dfrac{25}{0{\cdot}2} \bumpeq 120.$

Five figures are retained in the working.

$$\begin{array}{r|r}
 & 12037 \\ \cline{2-2}
2\cancel{1}\cancel{0}\cancel{8}\cancel{1} & 25376 \\
 & 4295 \\
 & 79 \\
 & 16
\end{array}$$

**Result,** 120·4.

**Explanation.**—The first figure in the quotient is **1** and the first remainder 4295.

**Cross out the last digit, 1,** in the divisor.

2108 into 4295 goes twice. Write **2** as second digit of quotient, and say:

$2\times 1 \quad =2$. This is nearer 0 than 10; ∴ carry 0.
$2\times 8 \quad =16$ and **9** are 25; write **9** and carry 2.
$2\times 0+2=2$ and **7** are 9; write **7**.
$2\times 1 \quad =2$ and **0** are 2.
$2\times 2 \quad =4$ and **0** are 4; there is nothing to write.

**Now cross out the last digit, 8,** of the divisor.

210 into 79 will not go. Write **0** as next figure of quotient.

**Now strike out the 0** in the divisor.

21 into 79 goes 3 times. Write **3** as next figure of quotient.

Say, $3 \times 0 = 0$ ; carry 0.
$3 \times 1 = 3$ and **6** are 9 ; write **6**.
$3 \times 2 = 6$ and **1** are 7 ; write **1**.

**Strike out the 1** in divisor.

$2\cancel{1}$ into 16 goes 7 times. Write **7** as fifth figure of quotient.

The rough calculation being 120 and the figures of the quotient being 12037, the **result**, correct to four significant figures, is 120·4.

Ex. 3. *Evaluate* $\dfrac{0{\cdot}0014975}{3{\cdot}1416}$, *correct to four significant figures.*

$$\frac{0{\cdot}0014975}{3{\cdot}1416} \triangleq \frac{0{\cdot}0015}{3} \triangleq 0{\cdot}0005.$$

$$\begin{array}{r|r} & 47667 \\ \hline 3\cancel{1}\cancel{4}\cancel{1}\cancel{6} & 149750 \\ & 24086 \\ & 2095 \\ & 210 \\ & 22 \end{array}$$

**Result,** 0·0004767.

*Note.*—Five figures being retained in both divisor and dividend, it will be found that the divisor 31416 will not go into the dividend 14975 ; therefore the latter is written 149750.

## EXERCISES XVII.

1. Divide 82·72 by 3·641, correct to three significant figures.
2. Divide 67·035 by 0·4372, ,, ,, ,,
3. Divide 0·006704 by 437·2, ,, ,, ,,
4. Divide 0·0078 by 4156, ,, ,, ,,
5. Divide 80·42 by 753100, ,, ,, ,,
6. Divide 435·6 by 39·37, correct to the nearest unit.
7. Divide 1308900 by 8·7628, ,, ,, thousand.

Calculate, correct to four significant figures :

8. 39·78÷0·08997.
9. 89·574÷47895.
10. 0·0004508÷0·0051372.
11. 0·69564÷88·981.
12. 10·99÷3·1416.

Answer the following:

**13.** There are roughly 1·094 yards in 1 metre. How many cm. are there in 1 inch?

**14.** A point moves through 48·72 feet in 2·74 seconds. Find the speed in feet per second.

**15.** 7000 grains ≏ 453·6 grams. How many grains are there in 1 gram?

**16.** A prime mover is said to be of 1 horse-power when it can work at the rate of 550 ft.-lb. per second. Another unit of power is the watt, which is equivalent to 0·737 ft.-lb. per second. How many watts are there in 1 horse-power? How many horse-power are there in 1 watt?

**Multiplication or Division by 5, 25, 125.**—It will be found convenient to use the following rules, the reasons for which should be sufficiently obvious:

To multiply by 5, move the decimal point one place to the right and divide by 2.

To divide by 5, multiply by 2 and move the decimal point one place to the left.

To multiply by 25, move the decimal point two places to the right and divide by 4.

To divide by 25, multiply by 4 and move the decimal point two places to the left.

To multiply by 125, move the decimal point three places to the right and divide by 8.

To divide by 125, multiply by 8 and move the decimal point three places to the left.

Note that, as division is the inverse of multiplication, the various operations for dividing by 5, 25 or 125 are the exact inverses of those for multiplying by these numbers.

**Powers of Numbers.**—If $x$ is any number, $x$ times $x$ is written $x^2$, which is called the '**square of x**' or '**x squared.**' Compare p. 20.

In the same way $x \times x \times x$ is written $x^3$, read '**x cubed**' or '**x to the power of three.**'

'$x$ to the power of 4' is written $x^4$, and means the product of four $x$'s.

Ex. 1. *Shew, by writing in full, that* $a^3 \times a^2 = a^5$.

$$a^3 \text{ means } a \times a \times a.$$

$$a^2 \text{ means } a \times a.$$

$$\therefore\ a^3 \times a^2 \text{ means } a \times a \times a \times a \times a, \quad \text{which is written } a^5.$$

Ex. 2. *Shew that* $a^4 \div a^2 = a^2$.

$$a^4 \text{ means } a \times a \times a \times a.$$

$$a^2 \text{ means } a \times a.$$

$$\therefore\ a^4/a^2 \text{ means } \frac{a \times a \times a \times a}{a \times a}, \quad i.e.\ a \times a \text{ or } a^2.$$

## EXERCISES XVIII.

**1.** What does $a^4$ mean? What does $y^5$ mean?

**2.** Shew, by writing in full, that $x^3 \times x^4 = x^7$ and that $a^8 \div a^3 = a^5$.

**3.** Write in its simplest form $b^2 \times b^7$, $c^8 \div c^5$, $y^2 \times y^3 \times y^4$, $z \times z^4 \div z^2$.

**4.** Find, to three significant figures, the value of $x^3$ and of $\frac{1}{x^2}$ when $x$ is 2, 3·5 and 14·7.

**5.** Find, to four significant figures, the value of $\frac{xy^2}{z}$ when $x$ is 3·5, $y$ is 2·5 and $z$ is 1·3.

**6.** Find, to four significant figures, the value of $2x^2 - 3xy + y^2$,

(*a*) when $x = 1{\cdot}2$ and $y = 2{\cdot}4$,
(*b*) when $x = 3{\cdot}33$ and $y = 11{\cdot}52$.

**Square Root.**—The square of 2 is 4; conversely the **square root** of 4 is 2. Similarly the square root of 9 is 3. In general, if $y$ is the square root of $x$, then $y^2 = x$. The square root of $x$ is written $\sqrt{x}$, so that the statements

$$y^2 = x$$

and

$$y = \sqrt{x}$$

are practically identical.

The following table, giving the squares of numbers between 1 and 20, is convenient for reference and useful for calculating square roots:

| Number. | Square. | Number. | Square Root. |
|---|---|---|---|
| 1 | 1 | 1 | 1 |
| 2 | 4 | 4 | 2 |
| 3 | 9 | 9 | 3 |
| 4 | 16 | 16 | 4 |
| 5 | 25 | 25 | 5 |
| 6 | 36 | 36 | 6 |
| 7 | 49 | 49 | 7 |
| 8 | 64 | 64 | 8 |
| 9 | 81 | 81 | 9 |
| 10 | 100 | 100 | 10 |
| 11 | 121 | 121 | 11 |
| 12 | 144 | 144 | 12 |
| 13 | 169 | 169 | 13 |
| 14 | 196 | 196 | 14 |
| 15 | 225 | 225 | 15 |
| 16 | 256 | 256 | 16 |
| 17 | 289 | 289 | 17 |
| 18 | 324 | 324 | 18 |
| 19 | 361 | 361 | 19 |
| 20 | 400 | 400 | 20 |

Ex. 1. *Find two numbers between which lie the numbers* $\sqrt{12}$, $\sqrt{120}$, $\sqrt{0{\cdot}2}$, *and* $\sqrt{0{\cdot}0032}$.

From the table,

(1) $16 > 12 > 9$, *i.e.* $4^2 > 12 > 3^2$;

$\therefore$ 12 lies between $4^2$ and $3^2$, and $\sqrt{12}$ lies between 4 and 3.

(2) $121 > 120 > 100$, *i.e.* $11^2 > 120 > 10^2$;

$\therefore$ $\sqrt{120}$ lies between 11 and 10.

(3) $0{\cdot}25 > 0{\cdot}2 > 0{\cdot}16$, *i.e.* $0{\cdot}5^2 > 0{\cdot}2 > 0{\cdot}4^2$;

$\therefore$ $\sqrt{0{\cdot}2}$ lies between 0·5 and 0·4.

(4) $0{\cdot}0036 > 0{\cdot}0032 > 0{\cdot}0025$,

*i.e.* $(0{\cdot}06)^2 > 0{\cdot}0032 > (0{\cdot}05)^2$;

$\therefore$ $\sqrt{0{\cdot}0032}$ lies between 0·06 and 0·05.

Ex. 2. *Find the square root of* 40·96.

Since 40·96 lies between 49 and 36, $\sqrt{40{\cdot}96}$ will lie between 7 and 6.

**Rule.**—Mark off the digits of the number in pairs from the decimal point, thus forming the groups 40, 96.

Next find the number whose square is less than but nearest to 40, viz. 6. Write 6 above the first group. Draw a horizontal line above the number and a vertical line to the left, and place the square of 6, viz. 36, under the first group, thus:

$$\begin{array}{r|l}
\multicolumn{2}{r}{6\phantom{0\;96}} \\ \cline{2-2}
 & 40\;96 \\
 & 36
\end{array}$$

Now subtract 36 from 40 and bring down the next group, viz. 96. **Double** the partial answer, 6, and place the result, viz. 12, to the left of the vertical line and opposite the remainder 496, thus:

$$\begin{array}{r|l}
\multicolumn{2}{r}{6\phantom{0\;96}} \\ \cline{2-2}
 & 40\;96 \\
 & 36 \\ \cline{2-2}
12 & \phantom{0}4\;96
\end{array}$$

Now 12 goes into 49, 4 times. Place 4 next to 6 in the partial answer, and next to 2 in the divisor, thus:

$$\begin{array}{r|l}
\multicolumn{2}{r}{6\;4\phantom{\;96}} \\ \cline{2-2}
 & 40\;96 \\
 & 36 \\ \cline{2-2}
124 & \phantom{0}4\;96
\end{array}$$

Multiply the divisor, 124, by the last figure, 4, of the answer, and place the product under 496, thus:

$$\begin{array}{r|l}
\multicolumn{2}{r}{6\;4\phantom{\;99}} \\ \cline{2-2}
 & 40\;99 \\
 & 36 \\ \cline{2-2}
124 & \phantom{0}4\;96 \\
 & \phantom{0}4\;96 \\ \cline{2-2}
\end{array}$$

There is no remainder.

The significant figures of the required square root are 64, and the result is therefore 6·4, since the root lies between 6 and 7.

Ex. 3. *Evaluate, that is, find the value of* 428·49.

428·49 is approximately 400; it is less than $(21)^2$.

$\therefore\ \sqrt{428{\cdot}49}$ is approximately 20.

$$\begin{array}{r|r}
 & 2\;0\;\;7 \\ \hline
 & 4\;28\;49 \\
 & 4\phantom{\;28\;49} \\ \hline
407 & 28\;49 \\
 & 28\;49 \\ \hline
\end{array}$$

**Result**, 20·7.

**Contracted Square Root.**—In practice, it is only necessary to find the square root of a number to a limited number of figures. Thus, suppose a bar is to be forged square so that its cross-section contains 40 square inches. The side of the square is evidently $> 6$ and $< 7$, and can be found to any degree of accuracy as follows:

Ex. 4. *Find* $\sqrt{40}$, *correct to three significant figures.*

As in other contracted calculations, one more figure is retained in working. Thus, we find the square root of 40·00.

$$\begin{array}{r|r}
 & 6\;3 \\ \hline
 & 40\;00 \\
 & 36\phantom{\;00} \\ \hline
123 & 4\;00 \\
 & 3\;69 \\ \hline
 & 31
\end{array}$$

Having brought down the last group and found the last remainder, 31, as previously explained, we form the next divisor, 126, as usual, viz. by doubling the partial quotient. We then strike out the 6, and proceed as in contracted division, thus:

$$\begin{array}{r|r}
 & 63\;25 \\ \hline
 & 40\;00 \\
 & 36\phantom{\;00} \\ \hline
123 & 4\;00 \\
 & 3\;69 \\ \hline
12\cancel{6} & 31 \\
 & 25 \\ \hline
 & 6
\end{array}$$

**Result**, $\sqrt{40} \simeq 6{\cdot}33$.

Ex. 5. *Find, to three significant figures, the radius of the circle whose area is* 753·7 *sq. cm., it being given that the radius of any circle is obtained by dividing the area by* 3·142 *and taking the square root of the quotient.*

We have to evaluate $\sqrt{\frac{753{\cdot}7}{3{\cdot}142}}$ to three figures.

Approximate result is $\sqrt{\frac{750}{3}}$, *i.e.* $\sqrt{250}$, which lies between $\sqrt{256}$ and $\sqrt{225}$. The radius is therefore between 15 and 16 cm.

$$\begin{array}{r|r}
 & 23986 \\ \cline{2-2}
\cancel{3}\cancel{1}\cancel{4}\cancel{2} & 7537 \\
 & 1253 \\
 & 310 \\
 & 27 \\
 & 2
\end{array}$$

$$753{\cdot}7/3{\cdot}142 \bumpeq 239{\cdot}9.$$

$$\begin{array}{r|r}
 & 1\,54\,8 \\ \cline{2-2}
 & 2,39,90 \\
25 & 1\,39\phantom{90} \\
304 & 14\,90 \\
3\cancel{0}\cancel{8} & 2\,74 \\
 & 28
\end{array}$$

Radius $\bumpeq$ 15·5 cm.

## EXERCISES XIX.

**1.** Use the table (p. 49) to write down the squares of 0·4, 1·5, 800, 150, 0·19, 0·02, 0·0011, 70, 1·7, 0·016, 18000, 0·000014.

**2.** Write down the square roots of 1·69, 2·89, 25600, 2·56, 0·0256, 1·21, 0·16, 0·09, 0·0001, 2·25, 0·000225, 0·000169, 0·00000324.

**3.** Evaluate

$5x^2-3y^2+150z^2$, where $x=18$, $y=1{\cdot}2$ and $z=0{\cdot}09$.

**4.** Calculate, correct to three significant figures, $\frac{2a-3b}{p^2+2q^2}$, where $a=17{\cdot}31$, $b=0{\cdot}983$, $p=0{\cdot}19$ and $q=1{\cdot}6$.

**5.** Verify the identity $x^2-y^2=(x-y)(x+y)$, where (i) $x=1{\cdot}2$, $y=3$; (ii) $x=140$, $y=0{\cdot}2$; (iii) $x=16$, $y=0{\cdot}018$; (iv) $x=190$, $y=-1800$.

**6.** Evaluate $\sqrt{\{8^2+15^2\}}$ and $\sqrt{\{0{\cdot}6^2+0{\cdot}8^2\}}$.

**7.** Use the table (p. 49) to write down pairs of numbers between which the following numbers lie :

$\sqrt{220}$, $\sqrt{1{\cdot}90}$, $\sqrt{0{\cdot}0075}$, $\sqrt{250}$, $\sqrt{3220}$, $\sqrt{0{\cdot}1}$,

$\sqrt{40}$, $\sqrt{3876}$, $\sqrt{3{\cdot}876}$, $\sqrt{0{\cdot}03876}$, $\sqrt{11{\cdot}84}$, $\sqrt{50578}$.

**8.** Find the square roots of 676, 9·61, 302500, 38025, 0·4356, 0·015876.

**9.** Evaluate, to three significant figures,

$\sqrt{43}$, $\sqrt{0{\cdot}3}$, $\sqrt{3}$, $\sqrt{0{\cdot}00056}$, $\sqrt{3{\cdot}1416}$, $\sqrt{3{\cdot}162}$.

**10.** Find $\sqrt{72}$ to five significant figures. Prove your answer by contracted multiplication. (M.C.U.)

**11.** Find $\sqrt{0{\cdot}123}$ to seven places of decimals. Prove your result by dividing 0·123 by your answer, using contracted methods. (M.C.U.)

# CHAPTER IV.

## ELEMENTARY MENSURATION.

IT is assumed that the student is working through a book on Practical Drawing,* and knows the definitions and common properties of the simpler figures and solids. A few exercises on geometry are included in this chapter.

**Rectangle.**—A four-sided figure which has all its angles right angles is called a **rectangle.**

Fig. 10 shews a rectangle with the axes, or centre lines, X′OX, Y′OY used in drawing the figure; their point of intersection O is called the centre of the rectangle or, briefly, the centre. O is the point about which the figure would balance. The lines XOX′, YOY′ are **axes of symmetry**.

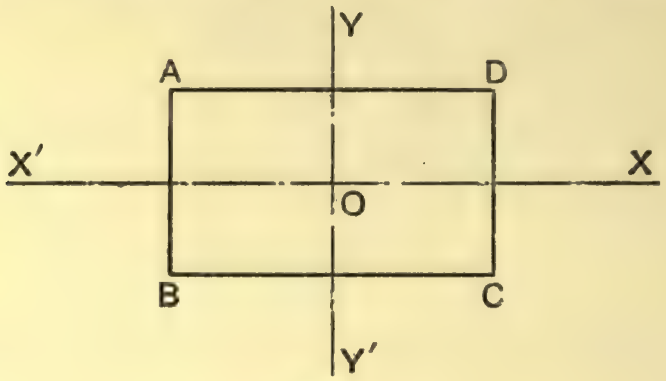


FIG. 10.

**Area of a Rectangle.**—**The unit of area** is a square, each side of which is of unit length. Thus, in the metric system, the unit of length is the centimetre; consequently the unit of area is a square, each side of which measures one centimetre. The area enclosed by the figure is called a square centimetre. Similarly the English unit of area is the square foot, square inch, or square mile, according to the amount of area to be measured.

---

* Such as *Practical Drawing*, by T. S. Usherwood (Macmillan), 2s.

## EXERCISES XX.

**1.** Draw a square of 1 ft. side, so that the surface covered by a square foot may be appreciated and compared with the square inch and square centimetre shewn in Fig. 11.

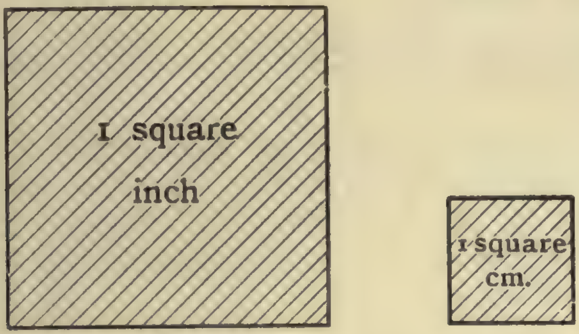


FIG. 11.

**2.** Divide each side of the 1 ft. square into 12 equal parts, and, by drawing lines parallel to the sides as shewn in Fig. 12, find how many square inches make one square foot.

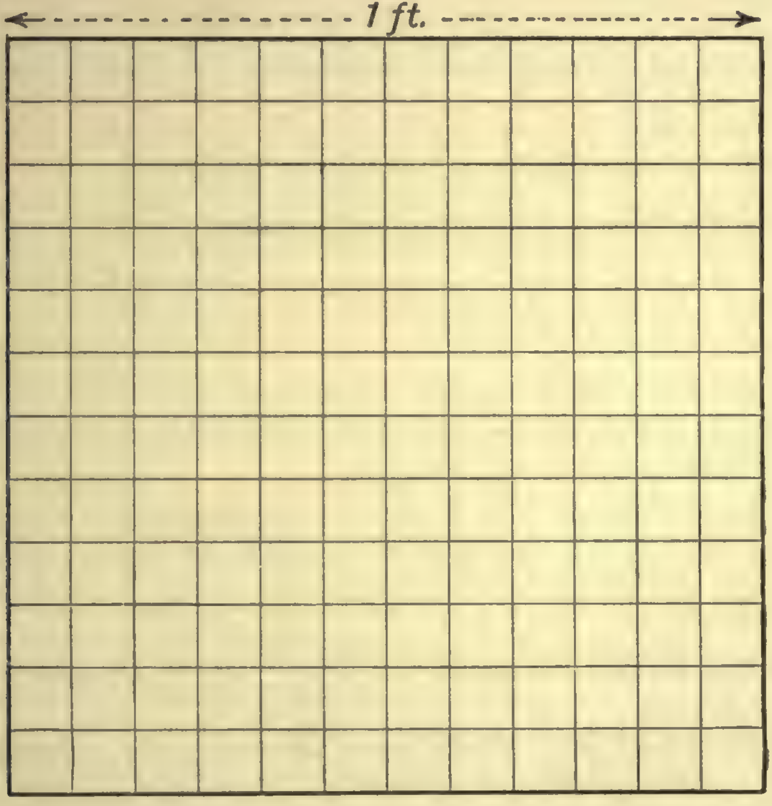


FIG. 12.

**3.** How many square cm. are there in a square m.?

**4.** How many square mm. are there in a square cm.?

**5.** How many square yards are there in a square mile? (1 mile = 1760 yd.)

### Examples on Areas.

Ex. 1. *How many feet super (i.e. square feet) are there in a 9″ board* L *in. long?*

Imagine the board marked off into strips 1″ wide by lines parallel to one end, as shewn in Fig. 13. The end strip evidently contains 9 sq. in. of area, since it contains 9 squares each 1″ × 1″. In the same way, every strip contains 9 sq. in. of area.

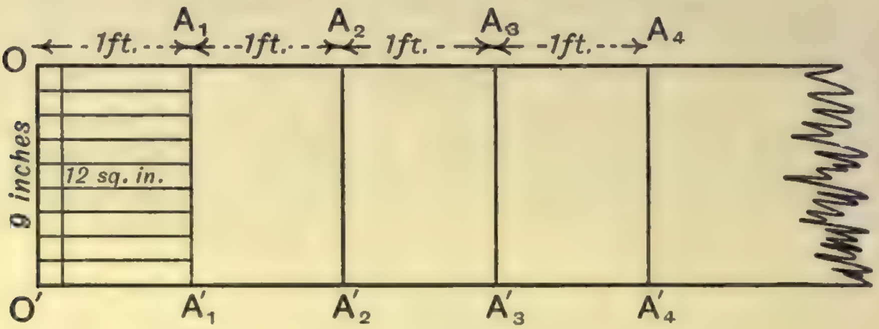


FIG. 13.

It is clear that a board 1 ft. long will contain twelve strips each of which contains 9 sq. inches of area.

Therefore a board 1 ft. long will contain $9 \times 12$ sq. in. of area,
" " 2 " " $2 \times 9 \times 12$ "
" " 3 " " $3 \times 9 \times 12$ "
" " 4 " " $4 \times 9 \times 12$ "

and so on, so that a board L ft. long will contain $L \times 9 \times 12$ sq. in. of area, where L is a whole number.

Now, since 1 sq. ft. = 144 sq. in., we must divide the number of square inches in the board by 144 in order to obtain the equivalent number of square feet.

Hence the board contains $\dfrac{L \times 9 \times 12}{144}$ sq. ft. of area.

The number of feet super in a 9″ board, L ft. long, is consequently $\dfrac{L \times 9}{12}$ or $\dfrac{3L}{4}$.

Ex. 2. *Find an expression for the area of a rectangle* L *ft. long and* w *ft. wide* (L *and* w *being whole numbers*).

Divide the rectangle into strips, one foot wide, by lines parallel to the side of length w ft. (Fig. 14.)

The number of square feet in the strip $AA_1B_1B$ is w.

(Since it is the same as the number of feet in AB.)

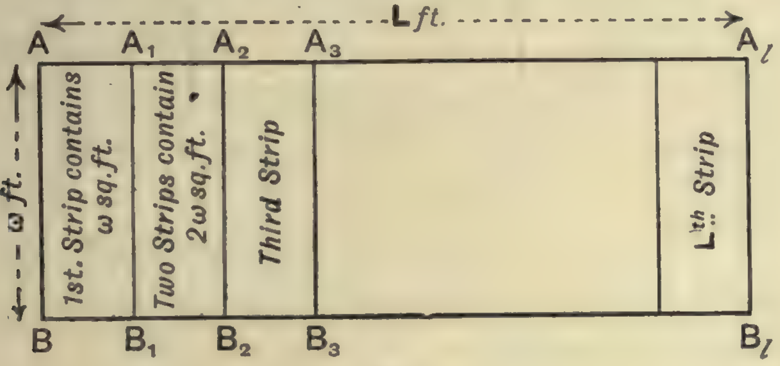


FIG. 14.

The number of strips will be L.

(Since it must be the same as the number of feet in AD.)

Hence the rectangle consists of L strips each containing w sq. ft.

∴ the area of the figure is wL sq. ft.

If the area of the rectangle be denoted by A sq. ft., we have

$$A = w \cdot L.$$

This is called a **formula** for the area of the rectangle.

It can be shewn that **the area of any rectangle is always obtained by multiplying its length and breadth together.**

Ex. 3. *Find a formula for the area of a square whose side is* s *in. long.*

A square is a rectangle whose length is equal to its width.

Now, for any rectangle, $A = wL.$

For the square, $w = L = s;$

$$\therefore\ A = s \cdot s$$

or $A = s^2$ is the formula for the area of any square.

Ex. 4. *Find a formula for the area of any triangle.*

Let ABC be any triangle. (Fig. 15.)

Through C draw a line parallel to the base AB, and from A and B draw AX, BY perpendicular to this line.

From C draw CP $\perp^r$ to AB.

By the construction, CYBP is a rectangle.

∴ CB, being a diagonal, bisects it.

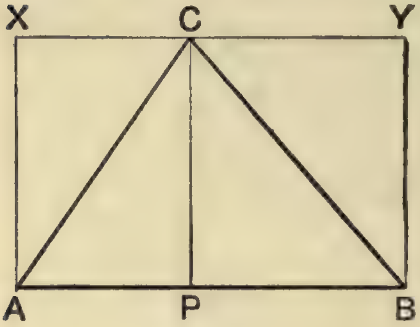


FIG. 15.

∴ $\triangle CPB = \frac{1}{2}$ rect. CYBP in area.

Similarly,

$\triangle CPA = \frac{1}{2}$ rect. CXAP in area.

∴ $\triangle CPB + \triangle CPA$

$= \frac{1}{2}$ rect. XYBA in area,

*i.e.* $\triangle ABC = \frac{1}{2}$ rect. XYBA.

But area of rect. XYBA = AB . BY.

= AB . CP.

∴ area $\triangle ABC = \frac{1}{2}$ . AB . CP.

In words, **the area of any triangle is one-half the base multiplied by the perpendicular height.**

Since area of a triangle = $\frac{1}{2}$ base × perpendicular height, it should be clear that triangles on the same base and between the same parallels are equal in area.

In Fig. 16, △s ABC, A′BC have the same bases, are between the same parallels, and consequently have equal areas. Mathematically, the △ ABC is said to have undergone a '**shear**,' being displaced into the position A′BC.

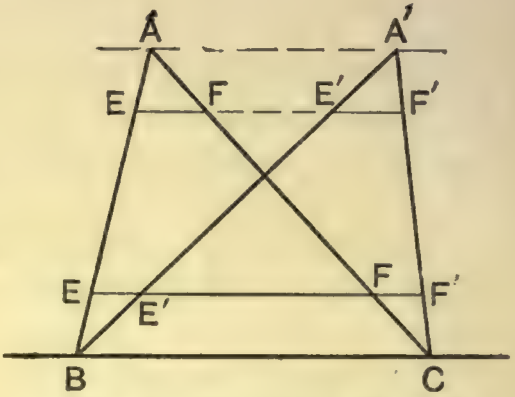


FIG. 16.—Illustrating 'shear.'

## EXERCISES XXI.

**1.** Draw a diagram to shew that 1 sq. yd. = $3^2$ sq. ft.

**2.** What is the area of a rectangle whose sides are

(i) 1·5″, 2″; (ii) 2·6″, 3″; (iii) 8″, 1′?

**3.** The area of a square is 1600 sq. cm. What is the length of its side?

**4.** The area of a rectangle is 36 sq. in. and one side is 4″ long. What is the length of the remaining side?

**5.** The area of a rectangle is A sq. cm. and one side is *a* cm. long. What is the length of the other side?

**6.** The sides of a rectangle are each four times as long as the sides of another rectangle. How many times is the smaller rectangle contained in the larger?

**7.** A rectangle 15 yd. by 9 yd. is drawn to a scale of 1 in. to 6 yd. What is the area covered by the drawing? Compare this area with that of the actual thing.

**8.** Find the area of an equilateral triangle of 2·5 inch side.

**9.** Find the area of the given figure (Fig. 17).

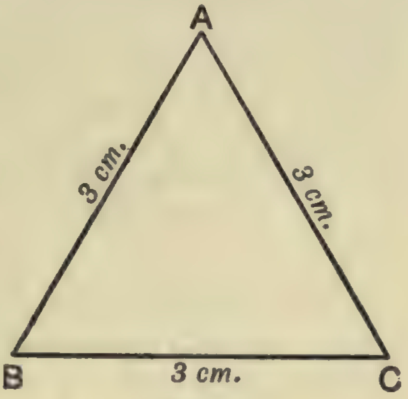


FIG. 17.

**10.** Fig. 18 is the plan of a field to a scale of 1 cm. = 50 yd. What is the area of the field?

(*Note.*—Draw a line to divide the field into two triangles.)

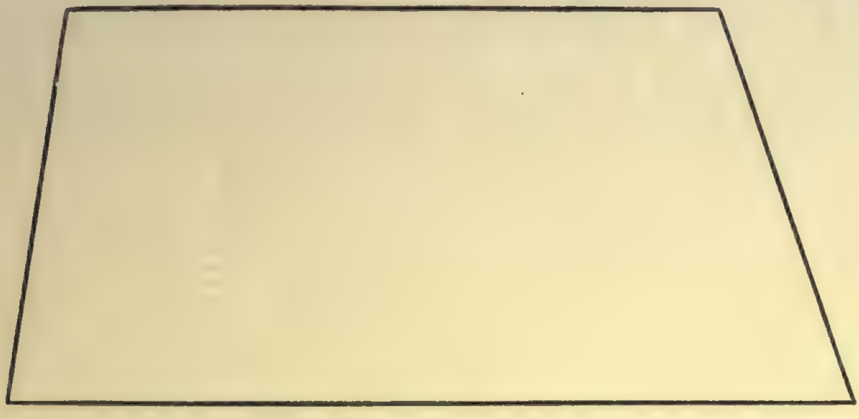


FIG. 18.

**11.** Prove that triangles on equal bases and between the same parallels are equal in area.

**12.** Shew that the length of any line parallel to AB, such as EF in the triangle ABC (Fig. 16), is unaltered by a shear.

**13.** Draw a triangle, sides 2·3″, 2·6″, 2·8″, and shear it into a right-angled triangle on a base 2·8″ long. Measure the side $\perp^r$ to the base, and deduce the area of the triangle.

**14.** Draw a triangle ABC, given $a = 7·5$ cm., $\hat{B} = 60°$, $\hat{C} = 45°$. Shear the triangle into an isosceles triangle on base BC, and measure the new angles B and C.

**15.** ABCD is a rectangle of paper whose sides are 3″ and 2″ long. Draw such a rectangle and the four creases that could be made by folding it so that a short side lay along a long side. Prove that the intersections of these four creases with one another are the corners of a square, and that the area of this square is ½ a square inch. (C.S.C.)

**16.** Draw a quadrilateral ABCD, given AB = 2·3 in., BC = 2·1 in., CD = 1·5 in., DA = 2·7 in., BD = 2·5 in. Shear the triangle BCD into one on the same base BD, having its vertex C′ on AB produced. Hence calculate the area of the quadrilateral.

**17.** Take a quadrilateral ABCD, given that AB = 4″, BC = 5″, CD = 6″, ∠ABC = 90° and ∠BCD = 75°. Find the area of the figure. If the figure represents an estate on the scale of 5″ to a mile, what is the area of the estate in acres? (C.S.C.)

(*Note.*—640 acres = 1 square mile.)

**Theorem of Pythagoras.**—To set off a right angle, carpenters frequently proceed as follows:

To draw through B (Fig. 19) a line perpendicular to AB, set off BX = 3 ft.; with centre X and radius 5 ft., describe an arc; with centre B and radius 4 ft., describe another arc cutting the first in C. CB is the required perpendicular.

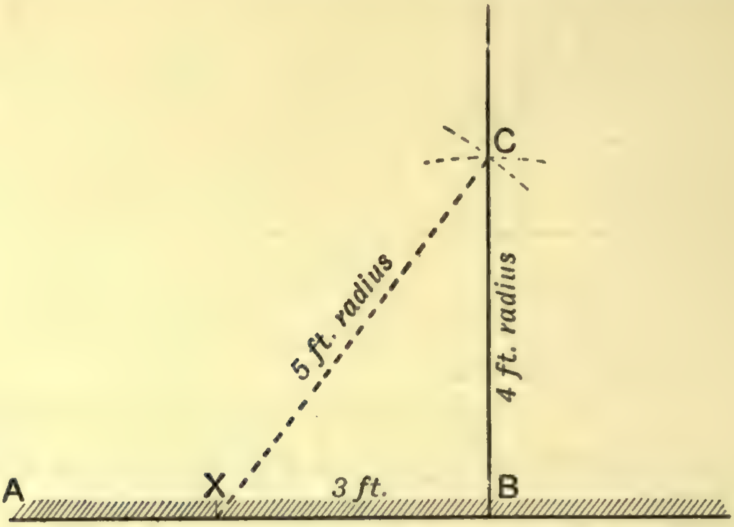


FIG. 19.—Illustrating the carpenter's method of setting off a right angle.

Note that the numbers 3, 4, 5 are such that

$$5^2 = 3^2 + 4^2.$$

In place of the lengths 3, 4 and 5 ft., any equimultiples may be used, *e.g.* 1·5, 2 and 2·5; 6, 8 and 10; 0·9, 1·2 and 1·5, etc. In every case the square of the largest number is equal to the sum of the squares of the other two, and if a triangle is constructed to have sides of such lengths, it will be right angled.

## EXERCISES XXII.

Draw a number of right-angled triangles, measure the sides and tabulate results as follows:

| No. of Δ. | Hypotenuse. | Other Sides. | | Square of Hypotenuse. | Sum of Squares of other Sides. |
|---|---|---|---|---|---|
| 1 | 5 | 3 | 4 | 25 | 9+16=25 |
| 2 | 1·5 | 0·9 | 1·2 | 2·25 | 0·81+1·44=2·25 |
| 3 | etc. | | | | |
| 4 | | | | | |
| 5 | | | | | |
| etc. | | | | | |

What may be inferred from the results?

It is important to remember that in *any* right-angled triangle, the square on the hypotenuse (that is the side opposite the right angle) is equal to the sum of the squares on the remaining sides. If the triangle ABC is right angled at C, then

$$c^2 = a^2 + b^2,$$

where $c$ is the side opposite the angle C, $b$ the side opposite the angle B, etc.

This fact was discovered by Pythagoras.

## EXERCISES XXIII.

**1.** Shew that the following triangles are right angled:

(*a*) $a=5·25$, $b=5·0$, $c=7·25$.
(*b*) $a=6·0$, $b=8·0$, $c=10·0$.
(*c*) $a=15$, $b=8$, $c=17$.
(*d*) $a=5$, $b=12$, $c=13$.
(*e*) $a=7·5$, $b=4$, $c=8·5$.
(*f*) $a=29$, $b=21$, $c=20$.

**2.** Shew experimentally that the triangle whose sides are $2x$, $x^2-1$, $x^2+1$, where $x$ has any value whatever, is always right angled.

## MISCELLANEOUS EXERCISES.

**1.** Fig. 20 represents a lawn with a footpath all round it. By means of the given scale measure the distance from A to D along the lines AB, BC, CD.

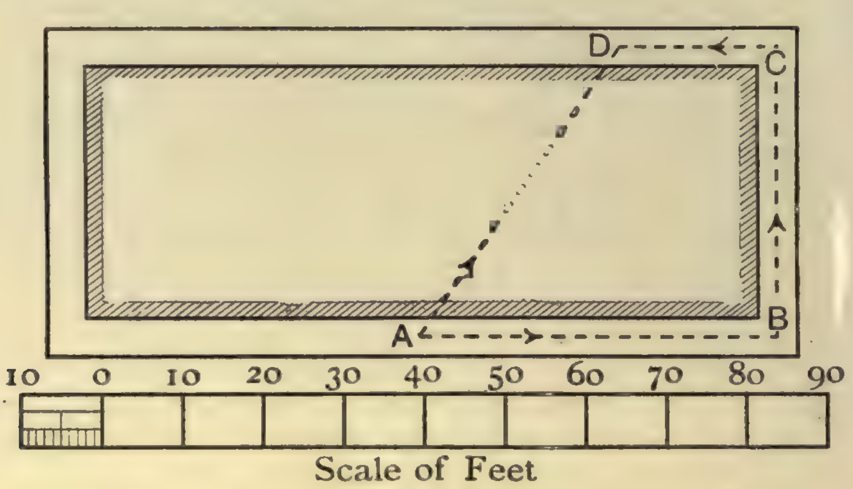


FIG. 20.

What would be the saving in distance by proceeding from A to D direct across the lawn?

Express the distances in feet and decimals of a foot.

(U.L.C.I.)

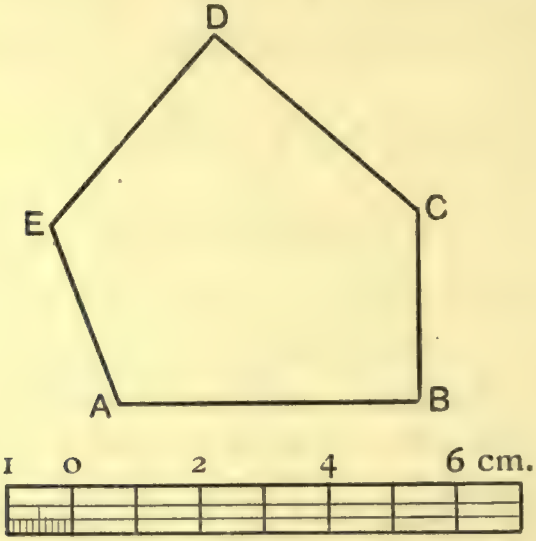


FIG. 21.

**2.** Write down the length of each side of the given figure ABCDE (Fig. 21) as measured on the given scale, and find the perimeter of the figure. (U.L.C.I.)

**3.** A triangle has two of its sides of length 2·7 and 2·1 inches respectively. If the angle opposite the shorter of these be 30°, how many distinct triangles are there satisfying the conditions? Construct them.

(Distinct triangles will here be understood to mean triangles which cannot be made to coincide by sliding them about in their own plane.)

If the length opposite the angle of 30° were 3 inches, would your result be altered? If so, how? (Matric.)

**4.** Construct a triangle having one side 2·35″, another 1·9″ and the angle opposite the latter 34°. Are there other triangles satisfying the same conditions, but having the third side different from that you have constructed? If so, construct them. (Matric.)

**5.** The side of a house 35 ft. long and 28 ft. high contains four windows each 3 ft. by 6 ft. What is the area of the brickwork? (U.L.C.I.)

**6.** The given dimensioned figure (Fig. 22) represents approximately an angle-bar section. On squared paper draw the figure full size. By dividing it into two rectangles, find its area in square inches. (U.L.C.I)

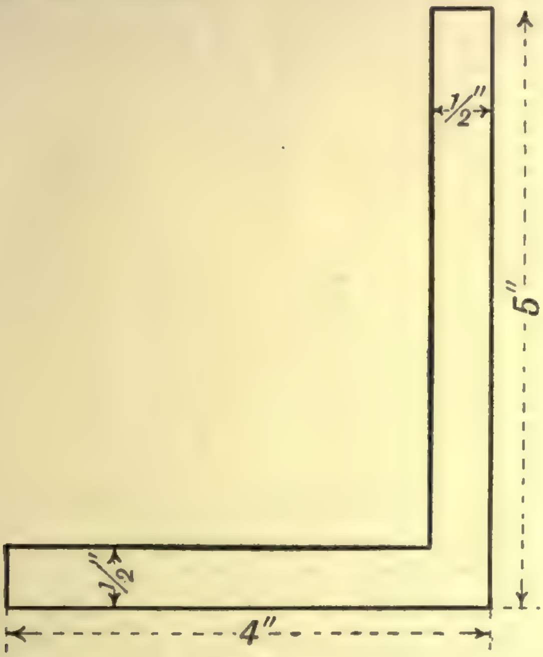


FIG. 22.

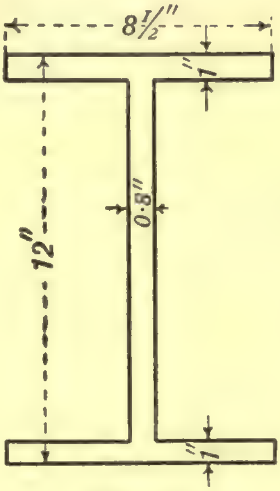


FIG. 23.

**7.** The given dimensioned figure (Fig. 23) represents approximately a rolled-beam section. On the squared paper draw the section half full size. Calculate the number of square inches in the section. (M.C.U.)

**8.** Calculate the area represented by the shaded portion of Fig. 24, of which a dimensioned sketch is given. On the squared paper draw the figure to the scale of $\frac{1}{2}$ in. = 1 ft. (U.L.C.I.)

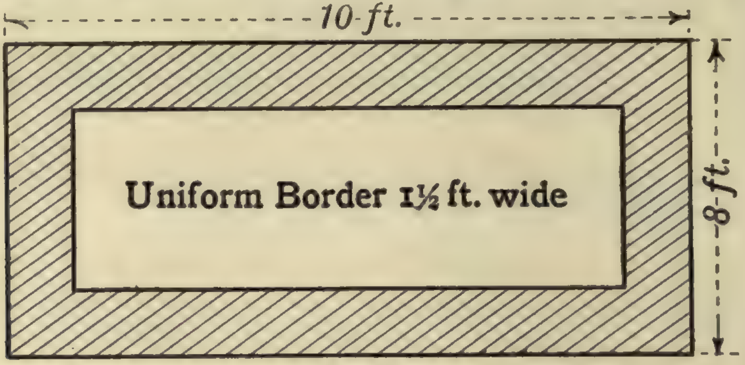


Fig. 24.

**9.** On the squared paper provided draw accurately a triangle having a base of 4″ and slant sides $3\frac{1}{2}$″ and $3\frac{1}{4}$″ respectively. From the vertex of the triangle draw a line at right angles to the base. Measure the altitude of the triangle. Calculate the area of the triangle, and on the same base draw a rectangle of equal area. (M.C.U.)

**10.** Fig. 25 represents a triangular plate drawn to the given scale. By means of the scale measure the base and the height of the triangle, and find the area of the plate in square inches. (U.L.C.I.)

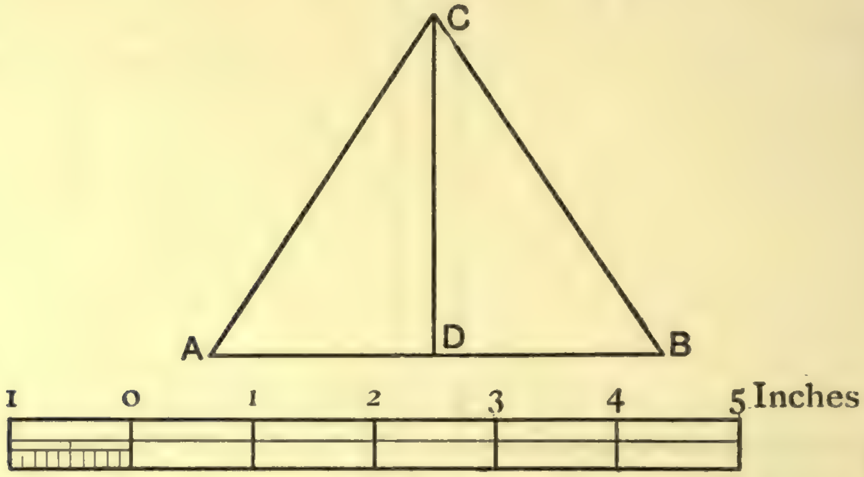


Fig. 25.

**11.** Construct a triangle, sides 3·4″, 4·0″ and 4·3″. Find its area by drawing and calculation.

**12.** Shew how to construct a triangle equal in area to a given quadrilateral. ABCD is a quadrilateral.

AB = 2 in., AD = BC = 2·5 in., CD = 3 in., BD = 4 in.

Construct this quadrilateral and find its area. (Matric.)

**13.** Construct a triangle with sides 3·5 in., 4 in. and 4·5 in.; upon the 4″ side construct an isosceles triangle of the same area, and find the area by measurement of your figure. (Matric.)

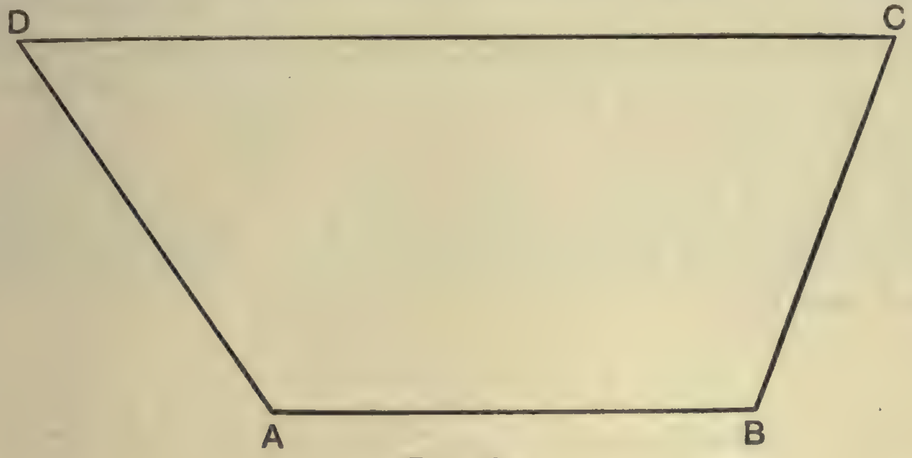


FIG. 26.

**14.** Using an inch rule, measure the sides of the given figure (Fig. 26), and write down the results. Also calculate the area of the figure by any convenient method. (M.C.U.)

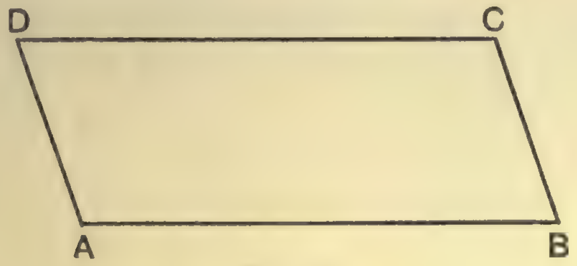


FIG. 27.

**15.** (*a*) With an inch rule measure the four sides of the figure ABCD (Fig. 27), and write down your results.

(*b*) Measure the four angles of the figure, write down your results, and add them together.

(*c*) Find the area of the figure by any convenient method.

(*d*) Find the length of the side of a square of equal area. (M.C.U.)

**16.** Draw accurately a triangle whose sides measure 2″, 2·5″ and 3″ respectively. Find the area of this triangle, and calculate the length of the side of a square of equal area. (M.C.U.)

**17.** Draw on squared paper a plan of a four-sided field ABCD from the following data:

$$AB = AD = 24 \text{ yards}; \ \hat{A} = 90^\circ; \ BC = CD = 28 \text{ yards}.$$

Estimate its area in square yards. (M.C.U.)

**18.** Draw a line 10 cm. long. Divide it into seven equal parts. Draw a line equal to four of these parts, and on it describe an equilateral triangle. Measure the altitude of the triangle, and hence determine its area. (M.C.U.)

**19.** Draw a circle of one inch radius on squared paper. In the circle inscribe a rectangle, one of whose sides measures 1·6 inches, and find the area of the rectangle. (M.C.U.)

**20.** The section of a handrail is shewn (Fig. 28). Draw this figure to the given dimensions, which are in mm. Shew the constructions by which the centres of the circular arcs are determined, and mark the junctions of the arcs. (B.E.)

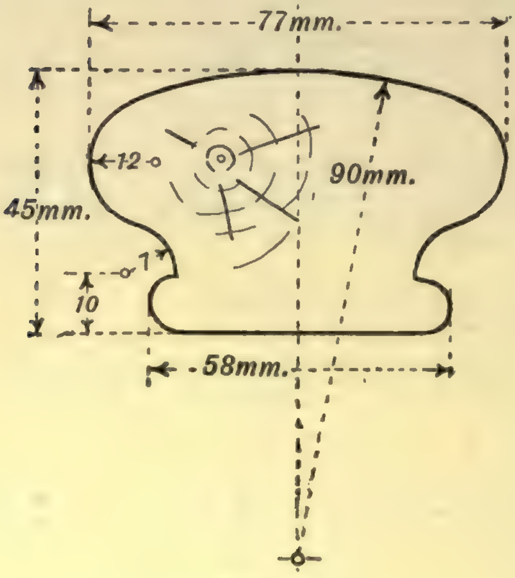


FIG. 28.

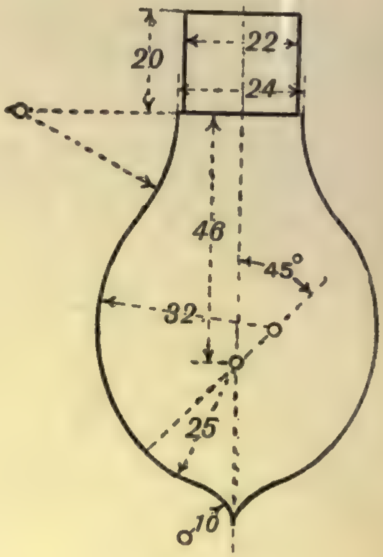


FIG. 29.

**21.** Draw the electric lamp (Fig. 29) to the dimensions given, which are in mm. Mark carefully the junctions of the several arcs. (B.E.)

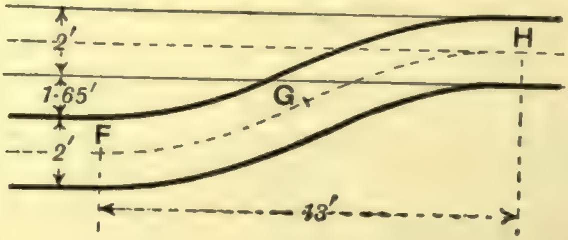


FIG. 30.

**22.** The figure (Fig. 30) shews a junction of rails for small waggons, the lines drawn dotted being midway between the rails. Set out the figure to a scale of 1 cm. to 1 foot, working to the given dimensions and not copying the diagram. Shew the construction for determining the centres of the circular arcs FG, GH, which are of equal lengths and radii. Indicate on the drawing the radii to which the several portions of the curved rails must be bent. (B.E.)

**The Unit of Volume** is defined as the space enclosed by a **cube**, each side of which is of unit length. The unit of volume is a cubic centimetre, a cubic foot, etc., according as the unit of length is one centimetre, one foot, etc.

**Volume of a Cuboid.**—A brick-shaped solid, that is one in which any three edges meeting at a point are perpendicular to one another, is called a **cuboid**. Its volume is determined by finding the number of unit cubes it contains.

Ex. 1. *Fig.* 31 *shews a cuboid, length* 10 *cm., breadth* 6 *cm., height* 8 *cm. Find its volume.*

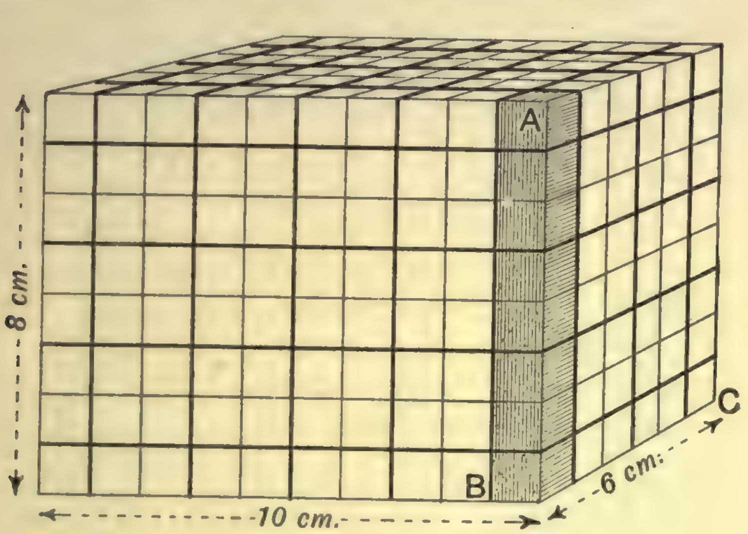


FIG. 31.

Dissecting into unit cubes, *i.e.* cubes occupying 1 c.c., it is seen that the shaded column contains 8 unit cubes; the right-hand slab ABC contains 6 such columns, while the cuboid contains 10 slabs like ABC. Hence the cuboid contains $8 \times 6 \times 10$ unit cubes, *i.e.* 480 c.c.

It should be clear that if the length, breadth and thickness of a cuboid are respectively $l$, $b$ and $h$ units of length, it could be dissected into $l$ slabs like ABC, each containing

$b$ columns like the one shaded in Fig. 31, and that each column would contain $h$ unit cubes, so long as $l$, $b$ and $h$ were whole numbers. The volume of such a cube is consequently obtained by multiplying together $l$, $b$ and $h$.

It can be shewn that the volume of any cuboid can be obtained by multiplying together the length, breadth and thickness, each expressed in terms of the same unit. Symbolically,

$$V = lbh.$$

Ex. 2. *The inside of an open tank measures* 2 *ft. long by* 2 *ft. wide by* $1\frac{1}{2}$ *ft. deep. It is made of lead* $\frac{1}{8}$ *inch thick. Lead weighs approximately* 0·42 *lb. per cubic inch. What does the tank weigh?*

The external dimensions of the tank are 24·25 in. long by 24·25 in. wide by 18·125 in. deep.

$\therefore$ the external volume is $24{\cdot}25 \times 24{\cdot}25 \times 18{\cdot}125$ cub. in.

$\therefore$ volume $\bumpeq 600 \times 18$ c. in. $\bumpeq 12000$ c. in.

$$\begin{array}{r}
\cancel{2}\cancel{4}\cancel{2}\cancel{5}\phantom{0} \\
2425 \\
\hline
485\phantom{0} \\
97\phantom{0} \\
5\phantom{0} \\
1\phantom{0} \\
\hline
\cancel{5}\cancel{8}\cancel{8}\phantom{00} \\
18125 \\
\hline
588\phantom{00} \\
470\phantom{00} \\
6\phantom{00} \\
1\phantom{00} \\
\hline
1065\phantom{00}
\end{array}$$

The volume enclosed

$= 24 \times 24 \times 18$ cub. in.

$\bumpeq 12000$ cub. in.

$$\begin{array}{r}
24\phantom{0} \\
24 \\
\hline
48\phantom{0} \\
96 \\
\hline
57\cancel{6}\phantom{0} \\
18 \\
\hline
576\phantom{0} \\
461\phantom{0} \\
\hline
1037\phantom{0}
\end{array}$$

$\therefore$ external volume

$\bumpeq 10650$ cub. in.

$\therefore$ the volume enclosed

$\bumpeq 10370$ cub. in.

$\therefore$ the volume occupied by lead $\bumpeq 10650 - 10370$ cub. in.

$\bumpeq 280$ cub. in.

Each cubic inch of lead weighs 0·42 lb.

$\therefore$ the tank weighs $280 \times 0{\cdot}42$ lb.

$\bumpeq 300 \times 0{\cdot}4$ lb.

$\bumpeq 120$ lb.

$$\begin{array}{r} 28\cancel{0} \\ 42 \\ \hline 1120 \\ 56 \\ \hline 1176 \end{array} \qquad \therefore \textbf{ Result is } 117{\cdot}6.$$

Weight = 120 lb. to two significant figures.

## EXERCISES XXIV.

**1.** Make a dimensioned sketch of a cube of 1 in. edge.

How many such cubes would be required to fill completely a box whose inside dimensions are: length 6 in., width 4 in., height 3 in.? (U.L.C.I.)

**2.** A rectangular solid (Fig. 32) has a length *a*, breadth *b*, height *c*.

What is the volume of such a solid?

If $a=5''$, $b=3{\cdot}4''$, $c=2{\cdot}6''$, find the volume of the solid, and also the area of its largest face. (U.L.C.I.)

State carefully the units in which the volume and the area are expressed.

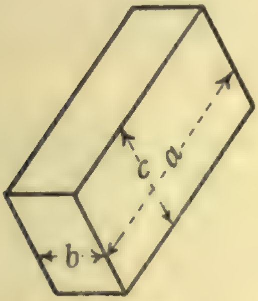


FIG. 32.

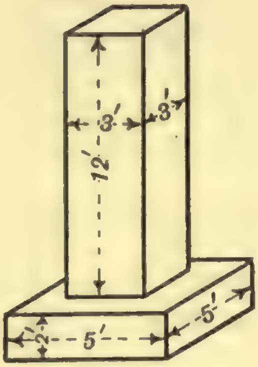


FIG. 33.

**3.** A brickwork pier is 3 ft. square and 15 ft. high. Find the number of cubic feet of brickwork it contains.

Make a dimensioned sketch of the solid. (U.L.C.I.)

**4.** The given figure (Fig. 33) is a dimensioned sketch of a concrete column and base.

Calculate the volume in cubic feet, and also the weight, if the concrete weighs 150 lb. per cubic foot (U.L.C.I.)

**5.** A rectangular block is 7·52 cm. long, 5·37 cm. broad and 2·53 cm. thick. Make a freehand sketch of the block, and calculate the area of one of the largest faces to three significant figures.

The material of the block weighs 7·5 grams per cubic centimetre. Find the total weight of the block. (M.C.U.)

**6.** A sheet of metal 9 cm. square weighs 96 grams. What will be the breadth of a rectangular sheet 22·5 cm. long and weighing 128 grams? (M.C.U.)

**7.** If a rectangular sheet of metal measuring 50 cm. by 40 cm. weighs 2000 grams, what will be the weight of a sheet measuring 30 cm. by 24 cm.? (M.C.U.)

**8.** A sluggish stream 10 ft. wide flows at the rate of 2 miles an hour. If the stream is 2 ft. deep, what volume of water flows past a given point per hour? (M.C.U.)

**9.** A rectangular tank is 6 ft. by 5 ft. How many more gallons of water will it hold if it is 5 ft. instead of 4 ft. deep? (1 cubic foot = 6·32 gals.) (M.C.U.)

**10.** An iron bar is 117 centimetres long and its cross-section is a square of which the side measures 9 millimetres. Find its weight to the nearest gram, supposing the iron to weigh 7·6 grams per cubic centimetre. (Matric.)

**11.** Two men measure a rectangular box; one finds its length, breadth and depth in inches to be 5·32, 4·15, 3·29. The other finds them to be 5·35, 4·17, 3·33. Calculate the volume in each case. What is the mean of the two? What is the percentage difference of either from the mean? (B.E.)

**12.** A swimming bath is 35 yd. long and 12 yd. wide. The depth of water varies uniformly from 3 ft. at one end to 6 ft. at the other (Fig. 34). Find the number of cubic feet of water the bath contains. (U.L.C.I.)

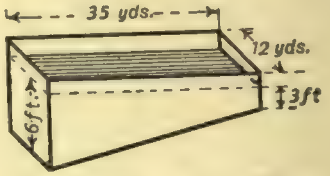


FIG. 34.

**13.** What is the cubic capacity of a barn 50 ft. long, 25 ft. broad and 12 ft. high, the roof of which is 6 ft. from the wall-plate to the ridge? (M.C.U.)

**14.** A rectangular sink is 3 feet long, 2 feet broad and 6 inches deep. What is the area of its inner surface? If this surface is covered with sheet lead $\frac{1}{32}$ inch thick, what is the volume of the lead? The specific gravity of lead being 11·36, what is the weight of the lead? (B.E.)

**15.** A square plate of wrought iron is 0·08 inch thick; it weighs 4 lb. What is the length of the side of the square? A cub. in. weighs 0·28 lb. (B.E.)

**16.** How many square feet of timber 1 in. thick are required to make a closed box with a base 4 ft. 6 in. × 3 ft. internal measurement which shall contain 400 gallons of water? (Take 277·3 cub. in. to gallon.) (Matric.)

**17.** What is the average depth of a rainfall in which 7 million litres of water fall on a square kilometre? (Matric.)

**18.** The areas of the faces of a rectangular solid are 285 sq. cm., 105 sq. cm. and 133 sq. cm. respectively. Find the cubic contents of the solid and the length of each edge. (M.C.U.)

# CHAPTER V.

## THE USE OF FORMULAE. ALGEBRAIC MANIPULATION. EQUATIONS.

In Molesworth's *Pocket Book for Engineers*, p. 394, a table is given, from which the following has been abstracted:

| Diam. | Circumferences. | | | | | | | | | | Diam. |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | **·0** | **·1** | **·2** | **·3** | **·4** | **·5** | **·6** | **·7** | **·8** | **·9** | |
| **36** | **113·0** | **113·4** | **113·7** | **114·0** | **114·3** | **114·6** | **114·9** | **115·2** | **115·6** | **115·9** | **36** |
| **37** | **116·2** | **116·5** | **116·8** | **117·1** | **117·4** | **117·8** | **118·1** | **118·4** | **118·7** | **119·0** | **37** |
| **38** | **119·3** | **119·6** | **120·0** | **120·3** | **120·6** | **120·9** | **121·2** | **121·5** | **121·8** | **122·2** | **38** |
| **39** | **122·5** | **122·8** | **123·1** | **123·4** | **123·7** | **124·0** | **124·4** | **124·7** | **125·0** | **125·3** | **39** |
| **40** | **125·6** | **125·9** | **126·2** | **126·6** | **126·9** | **127·2** | **127·5** | **127·8** | **128·1** | **128·4** | **40** |

The table is read as follows: Suppose it is required to find the circumference of the circle whose diameter is 37·4 units of length. Look down the left-hand column until 37 is reached, then across the row opposite 37 until the number under 0·4 is reached. This latter number is the required circumference, viz. 117·4 units of length.

How has this table been drawn up? If any circumference be divided by the corresponding diameter and the result be given to three significant figures, it will be found that in each case the quotient is 3·14. It is clear that if $c$ stand for the length of the circumference of any circle and $d$ stand for the diameter (measured in the same units as $c$), then

$$\frac{c}{d}=3{\cdot}14.$$

This is a formula connecting the circumference and diameter of any circle. It can be deduced mathematically from the known properties of the circle. The mathematical investigation really gives

$$c = 3{\cdot}1415926 \ldots d,$$

the number which multiplies $d$ having been evaluated to over 300 places of decimals! This number is conveniently denoted by the symbol $\pi$ (the Greek letter 'pi').

For all practical purposes $\pi$ may be taken as 3·142.

In Chapter IV. formulae for the area of a 9-inch board, for the area of any rectangle, for the area of any square and for the area of a triangle have been deduced. The formula for the area of a rectangle is $A = wl$, where $A$ sq. inches is the area of the rectangle, and $w$ in., $l$ in. are the lengths of its sides.

Ex. 1. *Deduce a formula for the distance traversed by a cyclist.*

Suppose a cyclist is riding at a steady rate of 8 miles per hour. It is clear that the distance he covers in one hour is 8 miles, in two hours is $2 \times 8$ miles, in three hours is $3 \times 8$ miles, and generally in $n$ hours is $n \times 8$ miles. Of course, $n$ need not be a whole number.

If the cyclist rides steadily at 9 miles per hour for $n$ hours the distance covered is $n \times 9$ miles.

If he rides steadily at 10 miles per hour for $n$ hours the distance covered is $n \times 10$ miles.

And generally if he rides steadily at $x$ miles per hour for $n$ hours the distance covered is $n \times x$ miles, where neither $n$ nor $x$ are necessarily whole numbers.

Hence, the formula connecting the distance a cyclist travels with the time he is cycling and the speed at which he cycles is

$$d = nx,$$

where $d$ miles is the distance travelled,
$n$ hours is the time taken
and $x$ miles per hour is the steady rate at which he cycles.

We should translate this formula into ordinary language as follows: the distance in miles = speed in miles per hour multiplied by the time in hours.

Ex. 2. **The principle of Archimedes** states that When a body is immersed in liquid it loses weight to the amount of the weight of liquid it displaces. Evidently, if it is totally immersed, the volume of liquid displaced is equal to the volume of the body. *A body weighs* W *gm. in air and* W′ *gm. in water. What is its density? What is its specific gravity?*

The weight lost is $W - W'$ grams.

$\therefore$ the weight of water displaced is $W - W'$ gm.

$\therefore$ the volume of water displaced is $W - W'$ c.c.

$\therefore$ the volume of the body is $W - W'$ c.c.

$\therefore$ the density is $\dfrac{W}{W - W'}$ gm. per c.c.,

and the specific gravity is $\dfrac{W}{W - W'}$.

Ex. 3. *A lump of metal*, S.G. 9, *weighs* 48 *grams in water. What is its weight in air?*

Let $w$ gm. denote the weight in air.

Then $w - 48$ gm. is the weight lost.

$\therefore$ $w - 48$ gm. = weight of water displaced.

$\therefore$ $w - 48$ c.c. = volume of water displaced.

$\therefore$ $w - 48$ c.c. = volume of body.

$$\therefore \text{S.G.} = \frac{w}{w - 48}.$$

But

$$\text{S.G.} = 9.$$

$$\therefore w/(w - 48) = 9.$$

In words, 'nine is the quotient of $w$ divided by $w - 48$.'

Therefore $w$ must be nine times $(w - 48)$.

$$\therefore w = 9 . w - 9 \times 48.$$

$$\therefore w = 9 . w - 432.$$

But

$$w = 9w - 8w.$$

$$\therefore 8w = 432.$$

$$\therefore w = 54.$$

The weight of the lump of metal is 54 gm.

## EXERCISES XXV.

**1.** It is known that if we divide the number of feet in the length of a pendulum by 32·2 and multiply the result by $4\pi^2$ (for the meaning of $\pi$, see p. 73), we obtain a number which is the square of the number of seconds in the time required for a complete oscillation. Express this by a formula, using $t$ to denote the number of seconds required for a complete oscillation, and $l$ to denote the number of feet in the length of the pendulum.

**2.** Suppose a man lifts a weight of P lb. through a vertical height of $h$ feet, and suppose the work required to do this is represented by the symbol W. It is known that $W = Ph$. Express this formula in words.

**3.** If G gallons of water fall through a distance of $h$ ft. in every second, the horse-power available is given by the formula $H = 0{\cdot}0182Gh$. Express this in words.

**4.** Express symbolically 'the sum of the reciprocals of two numbers is equal to the sum of the numbers divided by their product.' $\left[\text{The } \textbf{reciprocal} \text{ of } x \text{ is } \frac{1}{x}.\right]$

**5.** The weight of a body is W gm., its volume is V c.c. and its density is $\rho$ gm. per c.c. What is the formula for $\rho$ in terms of V and W?

**6.** If $\rho$ is the density of a body in gm. per c.c. and $w$ its weight in gm., what is the formula for its volume?

**7.** A body weighs 20 gm. in air and 15 gm. in water. What is its specific gravity?

**8.** A body weighs W lb. in air and $w$ lb. in water. What is its specific gravity?

**9.** In order to find the temperature in degrees Centigrade of a body whose temperature in degrees Fahrenheit is known, subtract 32 from the number of degrees Fahrenheit and divide by 9: the quotient is one-fifth of the number of degrees Centigrade. If F°, Fahrenheit, and C°, Centigrade, are corresponding temperatures, write down the formula connecting F and C.

**10.** The distance of the horizon in miles is approximately equal to 1·42 times the square root of the height of the observer's eye in feet. If $s$ miles is the distance of the horizon and $h$ feet the height of the observer's eye, what is the formula connecting $s$ and $h$?

**11.** If $n$ nautical miles equals the distance of the horizon and $h$ ft. equals the height of the observer's eye, then $n = 1{\cdot}23\sqrt{h}$. Express this in words.

**12.** If $W =$ weight of a given quantity of wet steam, $w =$ weight of water suspended in this steam, then $\frac{W - w}{W}$ is called the dryness fraction of the steam. Express this in words.

**Equations.**—In Ex. 3, p. 74, the formula S.G. $= \frac{w}{w-48}$ was employed. By using the known value, 9, for S.G., the *equation* $\frac{w}{w-48} = 9$ was obtained.

Speaking generally, if any formula whatever be taken, and the values of *all the symbols but one* in it are known, then, if these known values are substituted in it, the formula becomes what is called an equation in **one unknown quantity.** The *solution* of an equation involves some manipulation of the algebraic symbols used. Compare the working of Ex. 3 on p. 80.

Ex. 1. $CR = E$ *is a formula used in electrical engineering*, $C$ *being the current in amperes*, $R$ *the resistance in ohms and* $E$ *the electromotive force in volts. If the electromotive force of a dynamo is* 100 *volts and the total resistance of the circuit is* 4·5 *ohms, what is the current generated?*

Using the given formula and substituting 100 for $E$ and 4·5 for $R$, we have

$$C \times 4{\cdot}5 = 100\,;$$

$$\therefore\ 4{\cdot}5C = 100\,;$$

$$\therefore\ \frac{4{\cdot}5C}{4{\cdot}5} = \frac{100}{4{\cdot}5}\,;$$

$$\therefore\ C = \frac{100}{4{\cdot}5} \simeq 22{\cdot}2.$$

The current generated is 22·2 amperes nearly.

Ex. 2. *The diagram* (Fig. 35) *gives the dimensions of angle and 'Tee' iron. It can be shewn that the area* $A$ *of the cross-section in square inches is given by the formula*

$$A = at + bt - t^2.$$

*In a certain design* $A = 5$, $a = 4$, $t = 0{\cdot}5$. *What must be the depth of the beam* $b$?

Substituting the given values in the formula, the following equation to determine $b$ is obtained, viz.

$$5 = 4 \times 0{\cdot}5 + b \times 0{\cdot}5 - (0{\cdot}5)^2,$$

*i.e.*

$$5 = 2 + 0{\cdot}5b - 0{\cdot}25.$$

Subtracting 1·75 (*i.e.* $2-0{\cdot}25$) from each side, we have

$$3{\cdot}25=0{\cdot}5b.$$

Dividing both sides by 0·5, we have

$$6{\cdot}5=b.$$

The depth of the beam is therefore 6·5 inches.

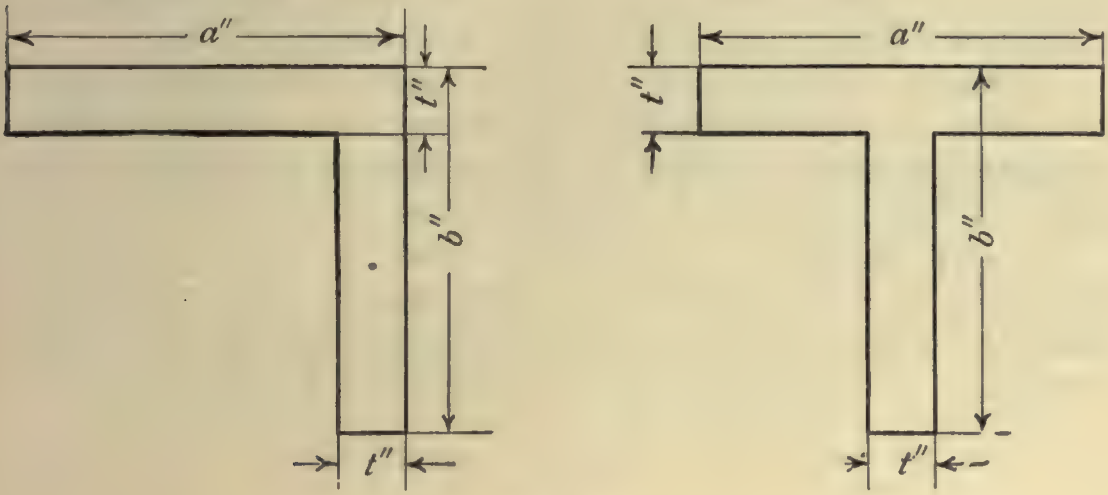


FIG. 35.

Ex. 3. *If a weight of* W *lb. is moving with a speed of* v *ft. per sec. in a circle of radius* r *ft., the force* F *lb. required to restrain it is given by the formula* $F=\frac{Wv^2}{r}$. *Find* r *if* $W=20$, $v=2$ *and* $F=96$.

Substituting, we have

$$96=\frac{20\times 2^2}{r};$$

$$\therefore\ 96=\frac{80}{r}.$$

Multiply both sides by r, and we have

$$96r=80.$$

Divide both sides by 96, and we have

$$r=\frac{80}{96}=\frac{8\times 10}{8\times 12}=\frac{10}{12}.$$

Therefore the radius of the circle is 10/12 ft., *i.e.* 10 inches.

In the preceding examples certain general properties of equations have been used. These may be summarised briefly as follows:

If $$A = B,$$
then
$$\text{(i)}\quad A + x = B + x,$$
$$\text{(ii)}\quad A - y = B - y,$$
$$\text{(iii)}\quad nA = nB,$$
$$\text{(iv)}\quad A/p = B/p,$$

where $x$, $y$, $n$ and $p$ are any numbers whatever, with the single exceptions that in (iii) and (iv) $n$ and $p$ cannot be zero.

In words: **In any equation, any number whatever may be added to, or subtracted from, either side, provided that the same number is added to, or subtracted from, the other side.** (This is equivalent to statements (i) and (ii).)

**In any equation, either side may be multiplied, or divided, by any number whatever except zero, provided that the other side is multiplied, or divided, by the same number.** (This is equivalent to statements (iii) and (iv).)

Ex. 4. *A lump of metal,* S.G. 7, *weighs* 36 *gm. in water. What is its weight in air?*

If $W$ gm., $w$ gm. be the weights in air and water respectively, then

$$\text{S.G.} = \frac{W}{W - w};$$

$$\therefore\ 7 = \frac{W}{W - 36}.$$

Multiplying both sides by $W - 36$, we have

$$7(W - 36) = W;$$

that is $$7W - 7 \times 36 = W.$$

Subtracting $W$ from and adding $7 \times 36$ to each side, we have

$$6W = 7 \times 36.$$

Dividing by 6, we have $W = 7 \times 6$;

∴ the weight of the metal in air is 42 gm.

Ex. 5. $E = C(R + r)$, *where* E *volts is the electromotive force;* C *amperes, the current;* R *ohms, the resistance of mains and lamps;* r *ohms, the resistance of the dynamo. Find* R *when* $E = 100$, $C = 20$, $r = 0{\cdot}5$.

We have $100 = 20(R + 0{\cdot}5)$.

Dividing both sides by 20, $5 = R + 0{\cdot}5$.

Subtracting 0·5 from both sides, $R = 4{\cdot}5$.

The resistance of mains and lamps is 4·5 ohms.

Ex. 6. *The area*, A *sq. in.*, *of a plank* w *in. wide and* L *ft. long is given by the formula* $A=12wL$. *How long is a* 15 *in. board which contains* 12 *ft. super?*

*Note.*—12 ft. super means 12 square feet, *i.e.* 12 × 144 sq. in.

Substituting, we have $12\times144=12\times15\times L$.

Dividing both sides by 12 × 15, we have

$$L=\frac{144}{15}=9{\cdot}6.$$

The board is, roughly, 9 ft. 7¼ in. long.

Ex. 7. *A straight line* 45 *in. long is divided into two parts, such that one part is half the other. What is the length of the longer part?*

Let $x$ in. be the shorter part.

Then $2x$ in. is the longer part.

Now, the short and long parts together make up the whole length 45 in.

$$\therefore\ x+2x=45\ ;$$

$$\therefore\ 3x=45\ ;$$

$$\therefore\ x=15\ ;$$

and $$2x=30.$$

Therefore the length of the longer part is 30 in.

## EXERCISES XXVI.

**1.** Two angles of a triangle are 20° and 40°. What is the other angle? Use the formula $A^\circ+B^\circ+C^\circ=180^\circ$.

**2.** Two angles of a triangle are 60° and 110°. What is the other angle?

**3.** ABC is an isosceles triangle, the vertex being A.

(*a*) If A is 75°, what is each angle at the base?

(*b*) If each angle at the base is 45°, what is the vertical angle?

**4.** A triangle is right angled and one of the acute angles is double the other. What are the other angles?

**5.** A straight line 63 cm. long is divided into two parts, one of which is 2½ times the other. What is the length of each part?

**6.** The distance S ft., which a body falls from rest in $t$ seconds, is given approximately by the formula $S=16t^2$. (*a*) How long does a body take to fall through 4 ft.? (*b*) How far does a body fall in 2 seconds?

**7.** For a certain weight of gas at constant temperature $P_1V_1=P_2V_2$, where $P_1$ is the pressure of the gas in lb. per sq. in. when the volume is $V_1$ cubic ft. and $P_2$ the pressure when the volume is $V_2$. The pressure is 100 lb. per sq. in. when the volume is 2 cubic feet. Find

(*a*) the pressure when the volume is 10 cubic feet,
(*b*) the pressure when the volume is 0·5 cubic feet,
(*c*) the volume when the pressure is 25 lb. per sq. in.,
(*d*) the volume when the pressure is 200 lb. per sq. in.

**Typical Problems.**—The following examples are types of problems which frequently occur:

Ex. 1. *Solve* $7x-3=6x+1$.

Subtracting $6x$ from, and adding 3 to, each side,

$$x=4.$$

Ex. 2. *Solve* $3y-\frac{2}{7}=y+1$.

Subtracting $y$ from, and adding $\frac{2}{7}$ to, each side,

$$2y=1+\tfrac{2}{7};$$
$$\therefore\ 2y \doteq 1{\cdot}286;$$
$$\therefore\ y \doteq 0{\cdot}643.$$

Ex. 3. *The sum of two numbers is* 7, *their difference is* 2·4. *What are the numbers?*

Let $x$ be the smaller number.

Then $x+2{\cdot}4$ is the larger number.

$$\therefore\ x+(x+2{\cdot}4)=7;$$
$$\therefore\ 2x+2{\cdot}4=7;$$
$$\therefore\ 2x=7-2{\cdot}4=4{\cdot}6;$$
$$\left.\begin{aligned}\therefore\ x&=2{\cdot}3\\ \text{and}\qquad x+2{\cdot}4&=4{\cdot}7.\end{aligned}\right\}$$

Therefore the numbers are 2·3 and 4·7.

*Verification.* $4{\cdot}7+2{\cdot}3=7$,

$$4{\cdot}7-2{\cdot}3=2{\cdot}4.$$

Ex. 4. *A man's age is six times that of his son. In 20 years time his age will be twice that of his son. What are their present ages?*

Let $x$ years be the present age of the son.

Then $6x$ years is the present age of the father.

In 20 years the father's age will be $6x+20$ years

and the son's age will be $x+20$ years.

But in 20 years the father will be twice the son's age.

$$\therefore\ 6x+20=2(x+20);$$

$$\therefore\ 6x+20=2x+40.$$

Subtracting 20 and $2x$ from each side,

$$4x=20;$$

$$\therefore\ x=5$$

and

$$6x=30.$$

Therefore the father's age is 30 years and the son's age is 5 years.

## EXERCISES XXVII.

Solve the equations:

**1.** $2x+7=x+12$.

**2.** $3x+10=x+4$.

**3.** $4x+4{\cdot}7=2-x$.

**4.** $7x+\frac{3}{8}=2x+1$.

**5.** $3(x+1)=2(x+2)+1$.

**6.** $\dfrac{x}{2}+\dfrac{3x}{4}=5$.

**7.** $3x-\dfrac{x}{3}=28-2x$.

**8.** $\dfrac{2x+3}{7}-x=\dfrac{x-11}{9}$.

Solve the following problems:

**9.** Divide a number 20 into two parts, so that one part is three times the other.

**10.** The sum of two numbers is 45·7 and their difference is 14·5. What are the numbers?

**11.** The sum of two numbers is 76 and their difference is equal to one-third of the greater. Find the numbers. (B.E.)

**12.** The area of a rectangle is 12 sq. in., and the shorter side is one-third the longer. What are the lengths of the sides?

**13.** Sixty pounds is divided between A, B and C, so that A has twice as much as B, and C has as much as A and B together. What does each receive?

**14.** A sum of money is divided between A and B, so that A has thrice as much as B. If A had received £3 less and B £3 more, then A would have had £4 more than B. What sum of money is divided?

**15.** Divide 95·5 into two parts, so that one may be as much above 55 as twice the other is below it.

**16.** What bonus must be given to each man, woman and child in a factory consisting of 80 men, 120 women and 60 children, if each woman is to receive two-thirds of what each man receives and each child half as much as each woman, and the total amount available is £1200?

**Brackets.**—In the preceding pages, use has occasionally been made of the sign ( ). This is called a bracket, and shews that **the figures or symbols included are to be treated as a whole.**

Thus, $(2+3)(4-2)$ means the product of the sum of 2 and 3, and the difference of 4 and 2.

Thus,
$$\begin{aligned}(2+3)(4-2)&=5\times 2\\&=10.\end{aligned}$$

$(2+3)+(4-2)$ means $5+2$, *i.e.* 7.

$(2+3)-(4-2)$ means $5-2$, *i.e.* 3.

$a(b+x)$ means that the sum of $b$ and $x$ is to be multiplied by $a$. If any numbers whatever be substituted for $a$, $b$ and $x$, it will be found that
$$a(b+x)=ab+ax.$$

## EXERCISES XXVIII.

**1.** Express in words $3(5+2)$, $(4-2)(3+5)$, $(4-2)(3-5)$, $(2+7)+(5-3)$, $(5+4-7)-(4+6)$, and find the value of each.

**2.** Find the value of $\dfrac{a-b}{a+b}$ when

(i) $a=2$, $b=3$. (ii) $a=5{\cdot}7$, $b=3{\cdot}8$.

(iii) $a=11{\cdot}8$, $b=9{\cdot}7$. (iv) $a=8{\cdot}4$, $b=-4{\cdot}8$.

**3.** Find the value of $\dfrac{\mathrm{E}}{\mathrm{R}_i+\mathrm{R}_e}$ when

(i) $\mathrm{E}=100$, $\mathrm{R}_i=2{\cdot}8$, $\mathrm{R}_e=3{\cdot}7$.

(ii) $\mathrm{E}=220$, $\mathrm{R}_i=1{\cdot}5$, $\mathrm{R}_e=2{\cdot}3$.

**4.** Shew that the following statements are true :

(i) $4+3-2=4+(3-2)=(4+3)-2.$

(ii) $4+2-3=4+(2-3)=(4+2)-3.$

(iii) $2-5+4=(2-5)+4=2-(5-4).$

**5.** Shew by substituting values for *a*, *b* and *c* that

(i) $a+b-c=a+(b-c)=(a+b)-c.$

(ii) $a-b+c=(a-b)+c=a-(b-c).$

(iii) $2(a-b)-3(b-c)=2a-2b-3b+3c.$

(iv) $-5(a-c+b)=-5a+5c-5b.$

**6.** Remove the brackets from

(i) $x-(y-z).$ (ii) $2x-3(y+z).$

(iii) $3(p+q)-2(p-q).$ (iv) $4(x+3)-3(x+4).$

(v) $5(x+y)+3(6-x).$ (vi) $3(y+z-2)-2(z-x+4).$

(vii) $15-(2z-4y+x).$ (viii) $4z-(3x+y)-(5z-2y).$

## MISCELLANEOUS EXERCISES.

**1.** Find the value of $4a^2+3ab-b^2$ when $a=3$, $b=2$. (U.L.C.I.)

**2.** You are told that $25{\cdot}4\times9{\cdot}065=230{\cdot}251$. By inspection only of the above, write down the results of the following :

$254\times90{\cdot}65,\ 2{\cdot}54\times{\cdot}9065.$ (U.L.C.I.)

**3.** The following dimensions refer to the French Transatlantic liner "Rochambeau." Express them in inches :

| | |
|---|---|
| Diameter of high pressure cylinder, - - | 880 mm. |
| „ intermediate „ - - | 1290 mm. |
| „ low pressure „ - - | 1460 mm. |
| Stroke, - - - - - - - | 1000 mm. |
| Diameter of crank shaft, - - - - | 410 mm. |
| The total heating surface in boilers, - - | 2491 sq. m. |

Express also in lb. per sq. in. the working pressure in the boilers, which is 14 kg. per sq. cm.

**4.** Multiply, by contracted methods, $3{\cdot}14159\times27{\cdot}65328$, correct to three decimal places. (U.L.C.I.)

**5.** Given $\frac{Wl}{4}=Mbd^2$, express each letter in terms of the others, and find the value of $b$ when $W=1000$, $l=120$, $M=600$, $d=5$. (U.L.C.I.)

**6.** Find the value to four figures of $\frac{27{\cdot}8\times 41{\cdot}5}{67}$. (U.L.C.I.)

**7.** Which of the two given quantities is the greater, $8\frac{1}{2}$ inches or 0·7 feet?

Express the difference as the decimal of a foot to three places. (U.L.C.I.)

**8.** The side of a house, 35 feet long and 28 feet high, contains four windows, each 3 ft. by 6 ft. What is the area of the brickwork? (U.L.C.I.)

**9.** Solve $4(x+2)-3(4-x)+24=34$, and find the value of $3x^2+4x-15$ when $x$ is 4. (U.L.C.I.)

**10.** Write down the values of (*a*) $4{\cdot}25\times 10$, (*b*) $0{\cdot}07\times 1000$, (*c*) $0{\cdot}53\div 10$, (*d*) $70{\cdot}24\div 100$. (U.L.C.I.)

**11.** Find the value of $y$ in the expression $y=\frac{10}{3}(2x-4)$ when $x=5$. (U.L.C.I.)

**12.** How many mm. are there in a length made up of 3 m. + 54 cm. + 8 mm.? (U.L.C.I.)

**13.** A bicycle with wheels 28 in. in diameter is wheeled along the ground until the wheels make 30 complete revolutions. Find in feet and inches the distance the bicycle has moved. (U.L.C.I.)

**14.** Find the area of a rectangle 7·5 cm. in length and 3·76 cm. in breadth to three significant figures only. (M.C.U.)

**15.** Find the value of $x$ in the equations below, and verify your answers:

(*a*) $3(x-2)-2(x+3)=4(2-x)$.

(*b*) $\frac{x}{3}-\frac{x}{4}=\frac{1-x}{6}$. (M.C.U.)

**16.** A brass tube 4 ft. in length weighs 5 lb. What length of tube will weigh $7\frac{1}{2}$ oz.? (M.C.U.)

**17.** Find the value of

$$\frac{4{\cdot}375+5{\cdot}006-6{\cdot}925+0{\cdot}05}{(4{\cdot}859-2{\cdot}7036-0{\cdot}9024)\times {\cdot}25}.$$ (U.L.C.I.)

**18.** Find the value of $x$ in each of the following equations:

(*a*) $7x+10=4x+19$.

(*b*) $5(3x-4)=40$. (U.L.C.I.)

**19.** Find the values by contracted methods of

$$389{\cdot}45673 \times 4{\cdot}78932,$$

$$45{\cdot}00632 \div 31{\cdot}956.$$

Reduce to decimal form $\frac{36}{925}$, $\frac{925}{36}$. (U.L.C.I.)

**20.** Wire is to be wound on a round tube $2\frac{3}{8}''$ diameter. If 151 turns all in one layer are required, how much wire will it be necessary to have? (U.L.C.I.)

**21.** The perimeter of a rectangle is 28 cm. and the length is $1\frac{1}{2}$ times the height. Find the dimensions of the rectangle, and calculate its area. (U.L.C.I.)

**22.** The length of a rectangle exceeds the breadth by $1\frac{3}{4}$ inches, and the perimeter measures 11·4 inches. Find the dimensions of the rectangle. (M.C.U.)

**23.** If the circumference of a wheel measures $3\frac{1}{7}$ times the diameter, find the number of revolutions made by a wheel $56''$ in diameter in travelling $1\frac{1}{2}$ miles.

What will be the number of revolutions of a wheel of half the above diameter if the distance to be travelled is 5 times as great? (M.C.U.)

# CHAPTER VI.

## THE USE OF SQUARED PAPER. GRAPHS.

**Tabular Statements.**—Tables or lists of numbers of various kinds frequently occur. In most cases there are *two* sets of associated numbers given or implied, from which certain information may be derived.

Ex. 1. *The table gives the number of hours of bright sunshine and the maximum temperature in degrees for the first two weeks of August in* 1912 *and* 1911 *at Kew, and the daily rainfall during the same period of* 1912.

| Day. | Sunshine in Hours. | | Max. Temp. in Degrees. | | Rainfall in Inches. |
|---|---|---|---|---|---|
| Aug. | 1912. | 1911. | 1912. | 1911. | 1912. |
| 1 | 7·2 | 5·9 | 66 | 86 | 0·01 |
| 2 | 7·5 | 11·5 | 66 | 83 | 0·08 |
| 3 | 2·0 | 6·9 | 67 | 76 | Nil |
| 4 | 2·4 | 10·7 | 70 | 80 | 0·38 |
| 5 | 8·2 | 1·8 | 65 | 79 | 0·01 |
| 6 | 1·3 | 10·8 | 61 | 79 | 0·24 |
| 7 | 2·9 | 12·8 | 68 | 84 | 0·12 |
| 8 | 3·0 | 13·6 | 66 | 89 | 0·11 |
| 9 | 3·4 | 13·3 | 65 | 100 | Trace |
| 10 | 2·4 | 11·5 | 65 | 85 | 0·37 |
| 11 | 3·5 | 8·3 | 65 | 87 | 0·12 |
| 12 | 0·2 | 12·2 | 60 | 89 | Trace |
| 13 | 2·9 | 12·6 | 62 | 91 | 0·02 |
| 14 | 2·3 | 13·5 | 63 | 88 | 0·16 |

This table is a record of the actual number of hours of sunshine and maximum temperature on any day between July 31st and August 15th in 1911 and 1912; it also shews the actual rainfall for the same period in 1912. If increased so as to cover a period of 5, 10 or more years, it would have its value for the meteorologist. As given, it will be seen that the hottest day in the first two weeks of August, 1911, was August 9th, when there were 13·3 hours of bright sunshine. The hottest day during the same period of 1912 was August 4th, when there were 2·4 hours of sunshine and 0·38 in. of rain.

Ex. 2. *The totals of goods carried for certain years on the Dortmund-Ems Canal are tabulated as follows:*

| Year. | Tons. |
|---|---|
| 1901 | 427,715 |
| 1903 | 754,337 |
| 1906 | 1,172,612 |
| 1908 | 1,363,705 |
| 1910 | 1,765,470 |

This table illustrates the development of traffic on the canal. In the brief period of a decade the traffic on this canal has more than quadrupled. In the first period of two years the increase was 326,622 tons; in the middle period of two years the increase was 191,093 tons; while in the last period of two years it was 401,665 tons.

The object of such tables is to preserve information in a compact form, so that it may be available for reference, especially for comparison. For example, exports during 1911 may be compared with those of 1901; the rate of increase of population from year to year may be shewn; the pressure of steam at 400° F. may be compared with that at 200° F., and so on.

In other circumstances, tables are useful for giving particular information: *e.g.* the weight of the standard hexagonal nut for a 1 in. bolt; the logarithm of a certain number; the temperature of steam at a pressure of 100 lb. per sq. in., etc.

**Graphical Representation.**—For some purposes tables are most convenient; but, in many cases, the information they contain is more conveniently presented in another way. Since at least two sets of numbers have to be represented, the alternative method suggested must shew two series of values which may or may not be similar in kind. Thus, on one hand, it may be convenient to shew the connection between inches and centimetres, which are both measures of length; on the other, it may be convenient to shew the connection between two dissimilar things, like the size of a saucepan and its price.

It is clear that the motion of a point from one place to another can be regarded as equivalent to two separate displacements along lines at right angles; one line is usually horizontal and in the sense west to east, the other is perpendicular to it and in the sense south to north. Thus, the displacement O to P (Fig. 36) is equivalent to the displacement ON along OX and the displacement NP parallel to OY; these displacements being, as it were, essentially different, may clearly be used to represent essentially different things.

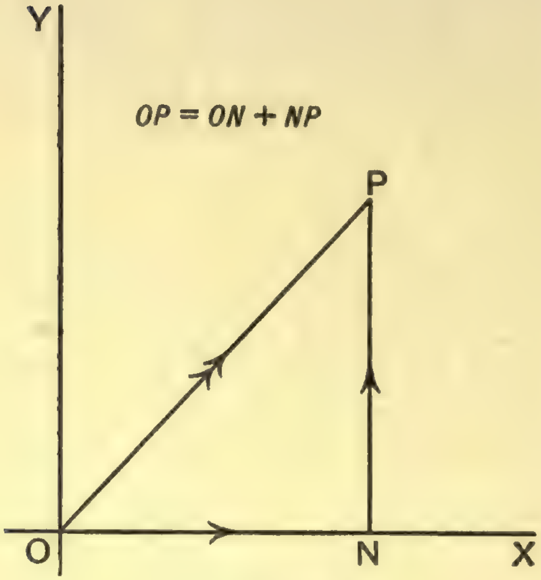


FIG. 36.

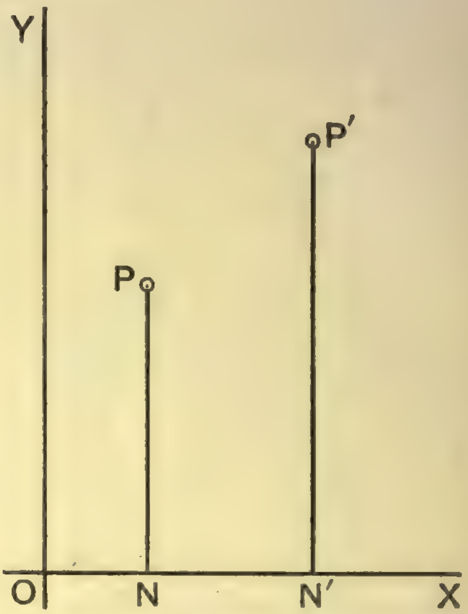


FIG. 37.

Thus, distances from OY—*i.e.* distances measured parallel to OX—may denote steam pressure in lb. per sq. in. on a certain scale, so that ON represents a definite pressure; while distances from OX—*i.e.* distances measured parallel to OY—may denote temperature in degrees Fahrenheit on a certain scale, so that NP represents a definite temperature. In this way the point P (Fig. 37) represents steam in a certain condition, at a certain definite temperature and pressure. Clearly then, a neighbouring point P′ would represent steam in another condition, viz. at a temperature represented by N′P′ and under a pressure represented by ON′.

Again, distances from OY—*i.e.* along OX—may represent time in years, so that the point N (Fig. 38) represents a certain year, while distances from OX—*i.e.* along OY—may represent imports of coffee in lb., so that NP represents a definite number of lb. of coffee. Under these conditions the point P indicates that so many lb. of coffee were imported in a certain year, while a neighbouring point Q shews that as many lb. of coffee as are represented by QR were imported in the year denoted by the point R.

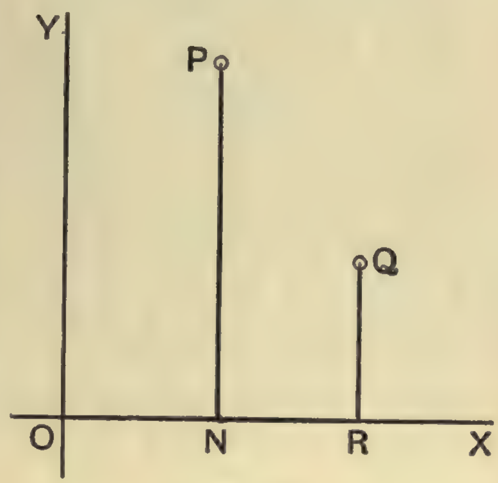


FIG. 38.

Here, then, is a convenient way of representing pictorially, or **graphically**, the information given by such tables as have been discussed, and certain examples are appended to shew the power and use of this method of illustration.

It is important to remember that, in general, the sets of numbers are essentially different in kind, and that in consequence there must be **two distinct scales**, one to each set of numbers, or variables. The scales must be such that the whole range of numbers is included conveniently in the diagram, that the points corresponding to the given numbers may be determined quickly; and, conversely, that the numbers corresponding to intermediate points may be determined readily.

**Use of Squared Paper.**—To save time and labour, **squared paper** should be used. This is paper covered with equidistant horizontal and vertical lines, every tenth line usually being marked distinctly, so that it is easy to measure off vertical and horizontal distances without using a graduated ruler. Various kinds of paper are available, but that divided into inch or cm. squares and sub-divided into tenths is generally used. With such decimally divided paper, it would generally be inconvenient to let one division represent such numbers at 3, 7, 13 etc., and it is best to arrange that each main division represents a number whose significant figures are 1, 2 or 5, as, in these

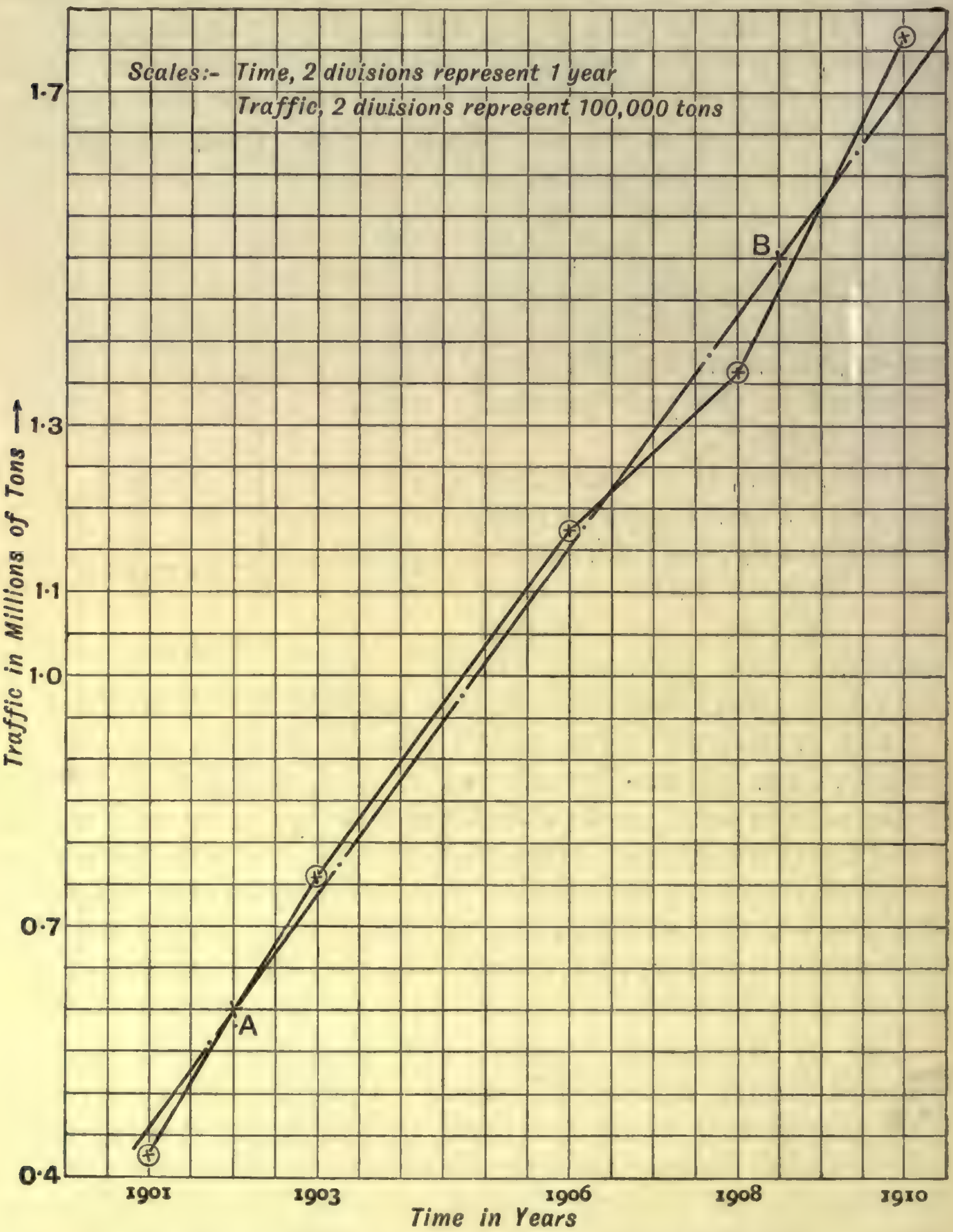


FIG. 39.—Growth of traffic on the Dortmund-Ems Canal.

circumstances, the numbers corresponding to sub-divisions of the paper are easily read off to almost any required degree of accuracy.

In every case it is important to **use as large a scale as possible** and to indicate every scale in plain figures on the diagram.

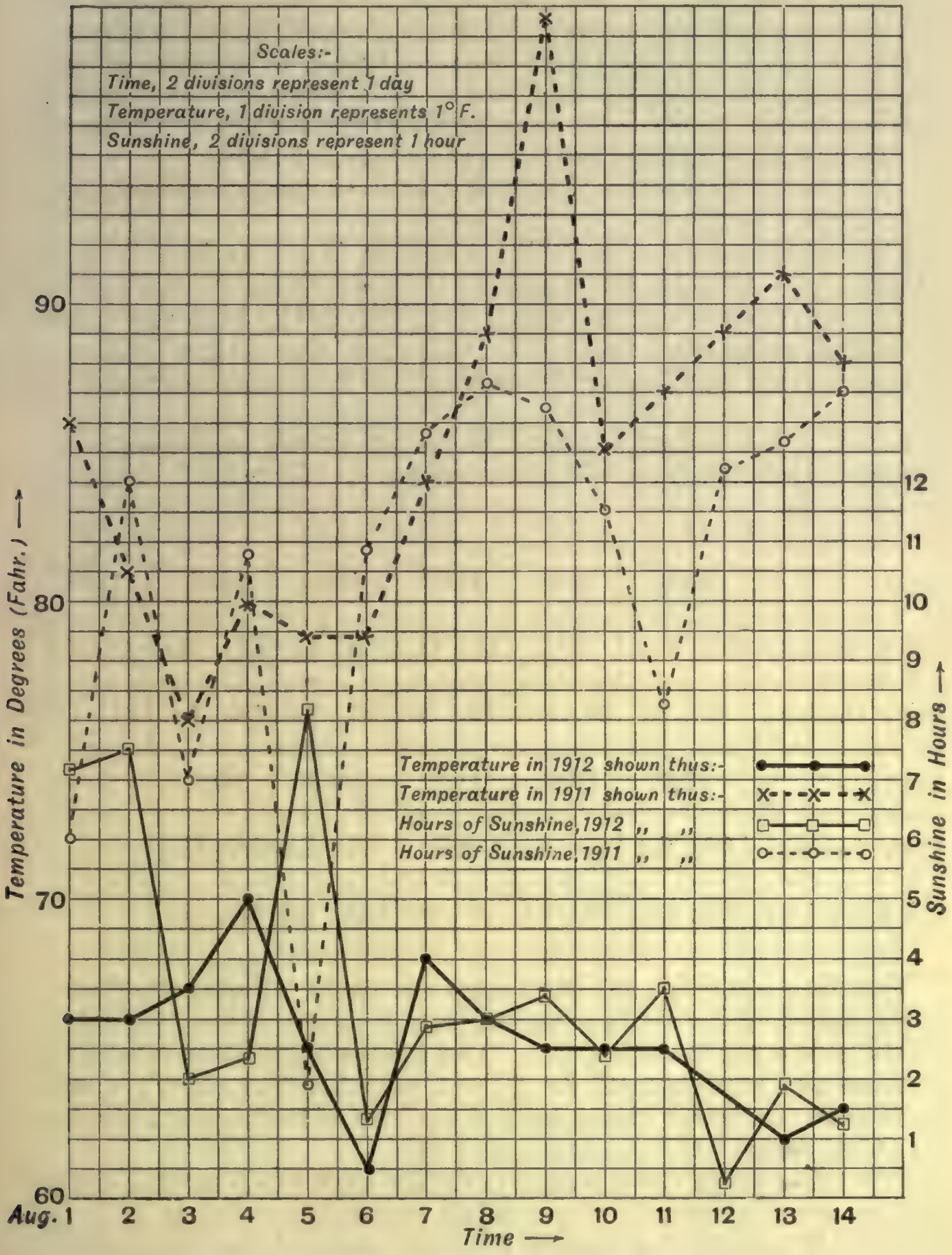


FIG. 40.—Representation of meteorological data.

Fig. 39 illustrates the table given in Ex. 2, p. 87. An inspection of the graph enables the information contained in the table to be appreciated at once. The plotted points lie nearly in a straight line, shewing that the traffic increased steadily. The average yearly rate of increase of traffic is found as follows : In the time interval represented by the horizontal distance between A and B, viz. 6·5 years, the increase of traffic in millions of tons, viz. that represented by the vertical distance between A and B, is 0·9 ; *i.e.* the traffic increased by 900,000 tons in 6·5 years, or at the rate of 138,000 tons per year approximately.

The plotted points are also joined to shew the actual variations from steady increase.

Fig. 40 illustrates the tables given in Ex. 1, p. 86. A comparison can readily be made of temperatures, variations in temperature, hours of sunshine, etc., for 1912, with the corresponding quantities for 1911. So, too, the graph shews a connection between hours of sunshine and maximum temperature.

Ex. 1. *The average yearly prices of wheat and barley per quarter from* 1890 *to* 1900 *are shewn in the table. Illustrate graphically.*

| Year - - | **1890.** | **1891.** | **1892.** | **1893.** | **1894.** | **1895.** |
|---|---|---|---|---|---|---|
| Price (wheat) | **31***s.* **11***d.* | **37***s.* | **30***s.* **11***d.* | **26***s.* **4***d.* | **22***s.* **10***d.* | **23***s.* **1***d.* |
| Price (barley) | **28***s.* **8***d.* | **28***s.* **2***d.* | **26***s.* **2***d.* | **25***s.* **7***d.* | **24***s.* **6***d.* | **21***s.* **11***d.* |
| Year - - | **1896.** | **1897.** | **1898.** | **1899.** | **1900.** | |
| Price (wheat) | **26***s.* **2***d.* | **30***s.* **2***d.* | **34***s.* | **25***s.* **2***d.* | **26***s.* **11***d.* | |
| Price (barley) | **22***s.* **11***d.* | **23***s.* **6***d.* | **27***s.* **2***d.* | **25***s.* **6***d.* | **24***s.* **11***d.* | |

The diagram (Fig. 41) illustrates the table. Times are plotted horizontally, 2 divisions representing 1 year ; prices vertically, 12 divisions representing 5*s.* The average prices of wheat per quarter are represented by points round which small circles have been drawn for clearness ; the prices of barley are indicated by small crosses for the same purpose.

The table gives the average price for each year, so that we do not get any information by joining consecutive points as in the figure. The full and dotted lines are only drawn to make it easy to distinguish the prices of wheat from the prices of barley. A continuous graph shewing the fluctuations throughout each year could be drawn if we had a table giving the daily or weekly price per quarter.

For convenience in representation, the price of barley in 1895 has been omitted from the diagram.

It will be seen that, on the whole, the two graphs rise and fall together, so that there is some kind of correspondence between the price of wheat and the price of barley. On the other hand, the price of either in any year is not connected with the time by any apparent law. Thus, if the average price of both for 1891 was not known, it would be impossible to guess it from the other readings. At the same time, if other statistics were plotted on the same sheet, it is quite possible that the graphs obtained would rise and fall with the graphs for wheat and barley, and so reveal certain connections.

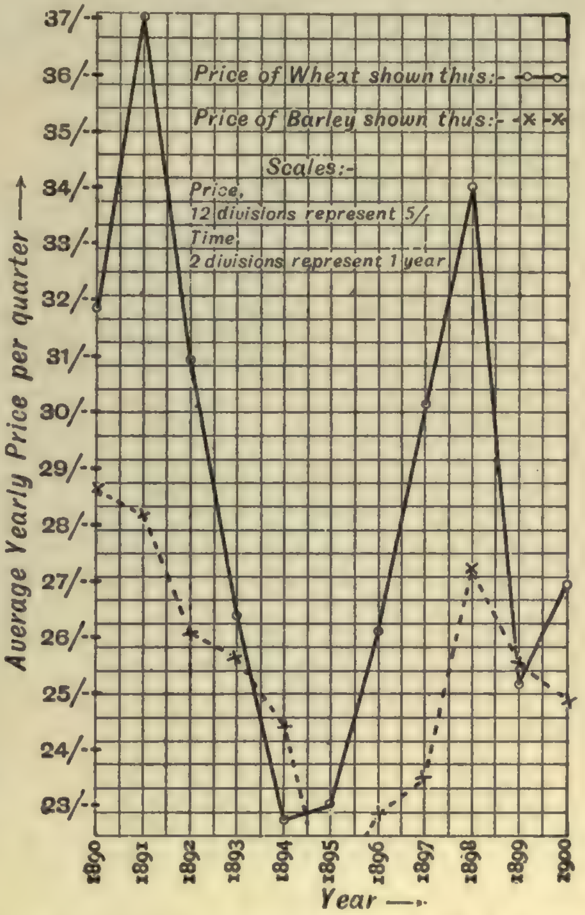


FIG. 41.—Variations in the prices of wheat and barley.

Ex. 2. *The following table, taken from the "Daily Mail Year-book," shews the percentages of unemployed members of trade unions in certain trades during the period from* 1897 *to* 1906. *Represent graphically.*

| Year | 1897. | 1898. | 1899. | 1900. | 1901. | 1902. | 1903. | 1904. | 1905. | 1906. |
|---|---|---|---|---|---|---|---|---|---|---|
| Building | 1·6 | 1·3 | 1·5 | 2·5 | 3·7 | 4·3 | 4·9 | 7·7 | 8·3 | 7·2 |
| Engineering | 3·8 | 3·8 | 2·5 | 2·8 | 3·8 | 5·1 | 4·9 | 7·0 | 5·1 | 3·0 |
| Shipbuilding | 7·6 | 4·7 | 2·3 | 2·5 | 3·7 | 8·2 | 12·0 | 14·0 | 11·9 | 7·6 |

Times are represented horizontally, the chosen scale being 2 divisions to represent 1 year. Percentages of unemployment are measured vertically, 2 divisions representing 1 %. The method of distinguishing the three trades is indicated on the diagram (Fig. 42).

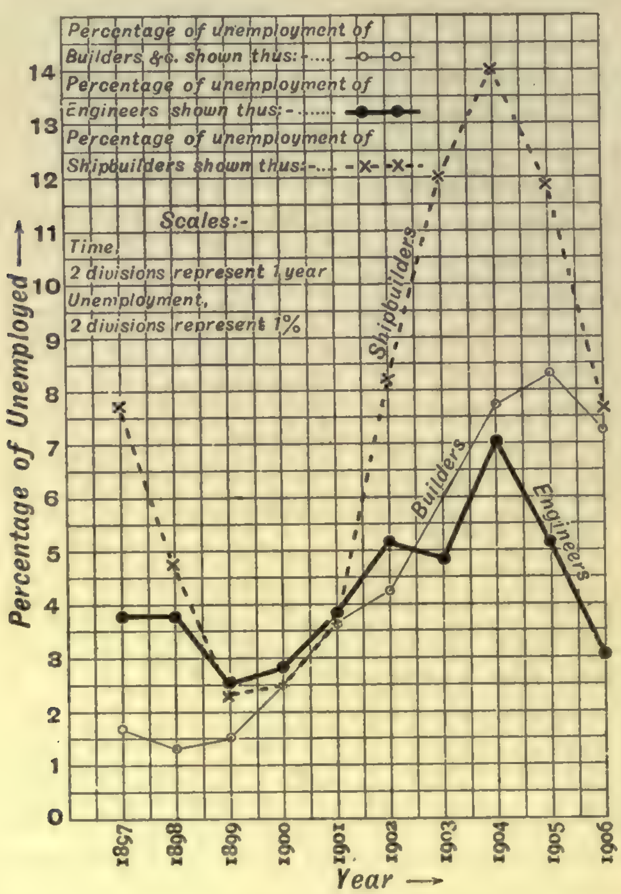


FIG. 42.—Unemployment statistics in certain trades.

Here, as in the last example, we have really a system of isolated points. No additional information can be gained by connecting them as on the diagram, but the plan makes it easier to distinguish the three classes of trades and *to trace the fluctuations of employment from year to year.*

Evidently certain influences are at work which affect the trades similarly, as the percentages of unemployment rise and fall more or less synchronously; but the connection between the engineering and shipbuilding trades is clearly closer than that between the building trades and the others; this is apparent if the full and dotted curves are compared.

Ex. 3. *The table shews approximately the distances of the horizon as seen from various heights above sea level. Find* (1) *the height at which the sea horizon is distant* 30 *miles* ; (2) *how far the eye can see from a height of* 1500 *ft.*

| | | | | | | |
|---|---|---|---|---|---|---|
| Height above sea level in feet - - - | 88·5 | 200 | 550 | 1200 | 2212 | 3186 |
| Distance of horizon in miles - - - | 10 | 15 | 25 | 37 | 50 | 60 |

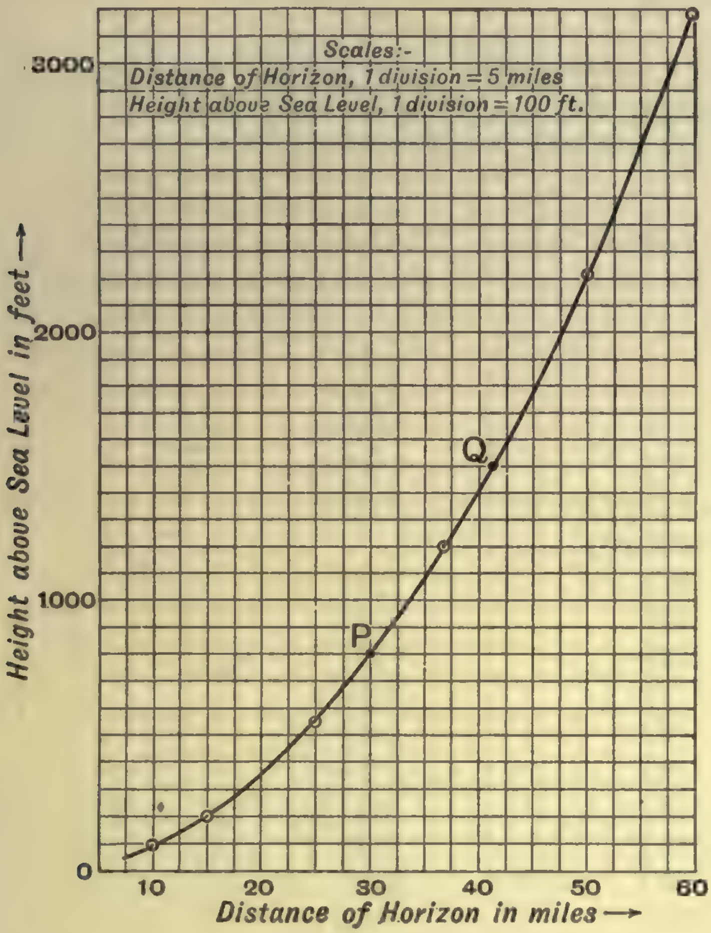


FIG. 43.—Connection between the distance of the horizon and the height of the observer's eye.

The distance of the visible horizon is plotted horizontally, the height above sea-level vertically (Fig. 43). In this case, a continuous curve may evidently be drawn through the plotted points, as there is a definite connection between the two variables. In other words, there is a definite *law* connecting the distance of the visible horizon and the height above sea-level, and, as a matter of fact, the numbers in the given table were calculated from the mathematical expression of the law.

To find the height of a cliff from which the sea horizon is 30 miles distant, look along the horizontal axis until the point marked 30 is reached, and then up to the curve. The height indicated by the point P is just under 800 ft.

(*N.B.*—The height at which the distance of the visible horizon is 30 miles is 797 feet.)

To find the distance of the sea horizon, as seen from a height of 1500 feet, look up the vertical axis until the point marked 1500 is reached, and then horizontally to the point Q. The distance is about 41 miles.

Ex. 4. *The table gives some of the properties of saturated steam* (*i.e. steam in contact with the water from which it is generated*). *Illustrate graphically.*

| Absolute Pressure in lb. per sq. in. | Temperature in degrees Fah. | Volume of 1 lb. in cubic feet. |
|---|---|---|
| 1 | 102·1 | 330·4 |
| 2 | 126·3 | 172·1 |
| 5 | 162·3 | 72·66 |
| 10 | 193·3 | 37·84 |
| 14·7 (atmospheric pressure) | 212 | 26·36 |
| 20 | 228 | 19·72 |
| 25 | 240·1 | 15·99 |
| 30 | 250·4 | 13·46 |
| 35 | 259·3 | 11·65 |
| 40 | 267·3 | 10·27 |
| 45 | 274·4 | 9·18 |
| 50 | 281·0 | 8·31 |
| 60 | 292·7 | 7·01 |
| 70 | 302·9 | 6·07 |
| 80 | 312·0 | 5·35 |
| 90 | 320·2 | 4·79 |
| 100 | 327·9 | 4·33 |

The scales chosen are indicated on the diagram (Fig. 44). It will be seen that two series of points are given: those indicated by crosses which connect pressures and temperatures; and those indicated by circles which connect pressures and volumes. The numbers given in the table are represented by the isolated points, but evidently the readings for intermediate points can be obtained from the graph, as the curves are *continuous*; there are no irregularities. Thus, at a pressure of 55 lb. per sq. inch, the temperature of saturated steam would be about 288° F., and the volume occupied by 1 lb. would be nearly 7·5 cubic feet. From these curves the connection between temperature and volume can be determined. Thus, at 260° F. the volume occupied by 1 lb. of saturated steam is 11·6 cubic feet nearly; for at 260° F. the pressure is just over 35 lb. per sq. inch, as shewn by the full curve, and when the pressure is 35 lb. per sq. inch the volume of 1 lb. is 11·6 cubic feet nearly, as shewn on the dotted curve. This is shewn by the points A and B.

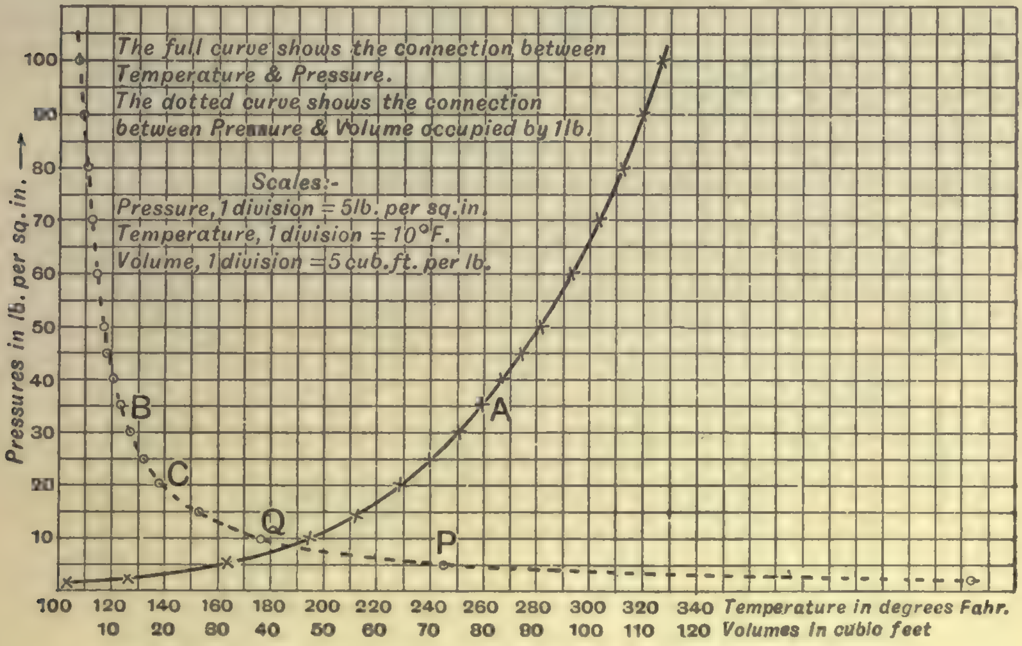


FIG. 44.—Graph of steam tables.

While the scales chosen are suitable for shewing the connection between temperature and pressure, they are unsuitable for shewing that between pressure and volume with any great degree of accuracy. For this latter purpose two graphs would be needed: one for pressures upwards of 20 lb. per sq. inch; and one for pressures less than 20 lb. per sq. inch. For the first of these, *i.e.* for that portion of the dotted graph to the left of the point C, the horizontal scale must be greatly magnified; for the second, *i.e.* that for points to the right of the point C, the vertical scale must be greatly magnified.

If three or four points on any continuous graph be taken, **the part of the graph containing those points can be magnified.** Thus, Fig. 45 illustrates the connection between pressure and volume of saturated steam for pressures between 20 and 40 lb. per sq. inch. It is a magnified view of the part of the diagram in the last figure between points B and C.

From the graphs in Fig. 44 certain general inferences may be drawn.

(i) As the temperature increases, the pressure increases, but more and more rapidly. Thus, a rise of temperature from 120° to 140° F. increases the pressure by about 1·5 lb. per sq. inch, whereas a rise from 290° F. to 300° F. increases the pressure by nearly 10 lb. per sq. inch. This is expressed mathematically by saying that the **rate of increase of pressure with temperature increases.** As we move to the right along the full curve, its slope or steepness increases.

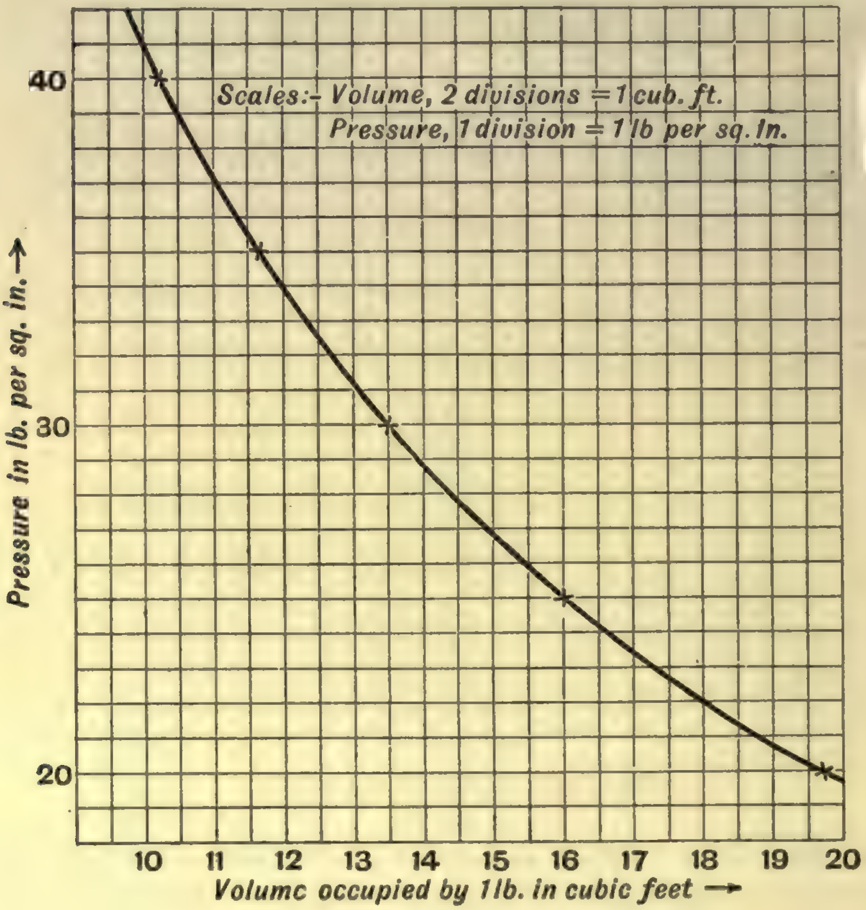


FIG. 45.—Magnified view of part of Fig 44.

(ii) As the pressure increases, the volume occupied by 1 lb. of saturated steam diminishes, but more and more slowly. Thus, an increase of pressure from 5 lb. per sq. inch to 10 lb. per sq. inch (*i.e.* a motion on the curve from P to Q) diminishes the volume occupied by 1 lb. by nearly 35 cubic feet ; while an increase of pressure from 80 lb. per sq. inch to 100 lb. per sq. inch only decreases the volume occupied by 1 lb. by about 1 cubic foot. We say that the **rate of decrease of volume with pressure diminishes,** or the rate of **increase of pressure with volume is negative, and its arithmetical value diminishes.** As we move to the right along the dotted graph, it slopes downward and gets less and less steep.

**Types of Graphs.**—There is a distinction between the types of numbers represented graphically, involving a corresponding distinction in the resulting graphs; some are *continuous*, some *discontinuous*. If, for example, the price of wheat throughout the year is shewn graphically, it is only the prices on particular days which are plotted; it cannot be said definitely that the price on some day omitted from the table was not quite different from any of the given prices, or even that the price may not have varied between wide limits on any day. Similarly, if the weekly average price of Consols for a range of years is graphed, it cannot be said that on a certain day the price did not suddenly jump from 75 to 79, or did not suddenly fall at another time from 82 to 80. In such cases the graphs consist merely of a series of separate points, and while variations are made more striking by joining the points, care must be taken not to read into the graph more than is justified by the data.

On the other hand, there are graphs which are really *continuous*, such as that illustrating the connection between the temperature and the pressure of steam; it is a pictorial representation of experiments on the condition of steam at *definite* pressures, not, of course, at *every* pressure; but it is reasonable to assume that every point on the continuous curve passing through the separate points shews the temperature of steam at corresponding pressures, and *vice versa*. Such graphs are usually regular as well as continuous, although there may be minor irregularities caused by errors in experimenting, by careless reading, or other sources of variation. There is another type of graph from which even such minor irregularities are absent, viz. those representing the value of algebraic expressions for different values of the variable. These are illustrated in the following examples:

Ex. 5. *Illustrate graphically the variation in value of the expression*

$$x(7{\cdot}5 - x),$$

*as* $x$ *increases from* $-5$ to $+10$.

| Values of $x$. | $7{\cdot}5 - x$. | $x(7{\cdot}5 - x)$. |
|---|---|---|
| $-5$ | 12·5 | $-62{\cdot}5$ |
| $-3$ | 10·5 | $-31{\cdot}5$ |
| $-2$ | 9·5 | $-19{\cdot}0$ |
| $-1$ | 8·5 | $-8{\cdot}5$ |
| 0 | 7·5 | 0 |
| 1 | 6·5 | 6·5 |
| 2 | 5·5 | 11·0 |
| 4 | 3·5 | 14·0 |
| 5 | 2·5 | 12·5 |
| 7 | 0·5 | 3·5 |
| 8 | $-0{\cdot}5$ | $-4{\cdot}0$ |
| 10 | $-2{\cdot}5$ | $-25{\cdot}0$ |

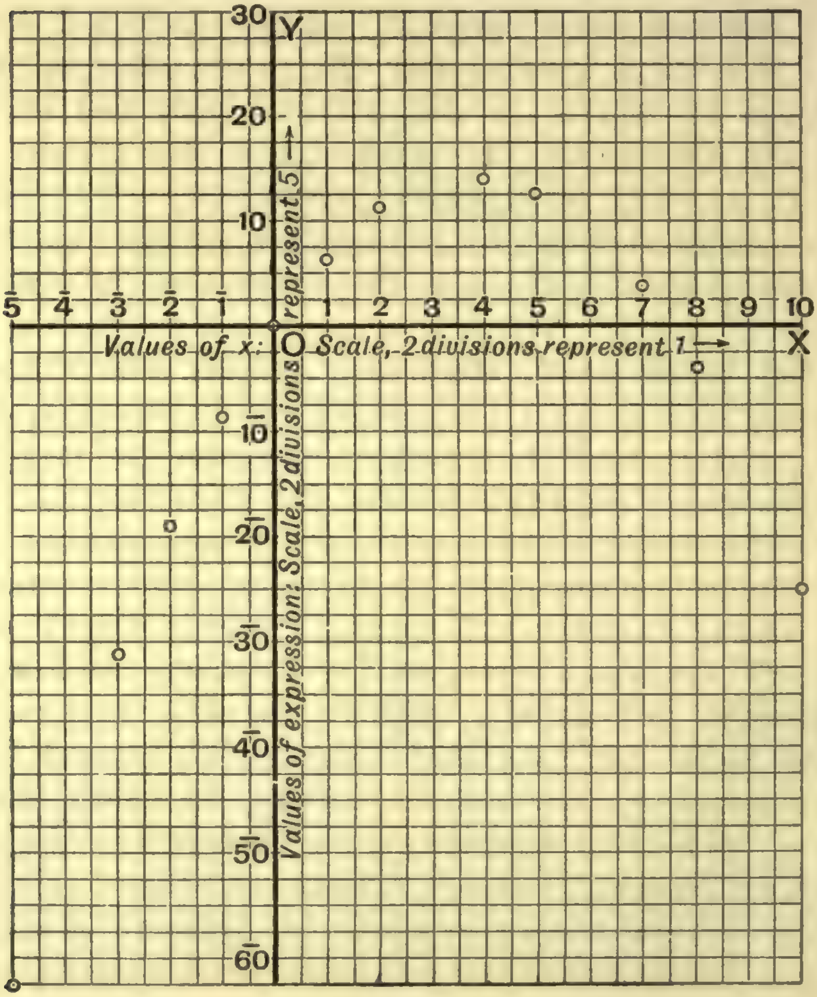


FIG. 46.

A negative value for $x$, viz. $(-5)$, is first selected and the value of the expression calculated. Other values of $x$ greater than the first are then taken, as shewn in the table, and the trend of the value of the expression can then be realised. Evidently for large $-^{ve}$ values of $x$ the expression is a large negative number; as $x$ increases the value of the expression increases (*i.e.* becomes a smaller negative number) until it reaches the value 0; it then becomes $+^{ve}$ and increases for some time, after which it decreases again, becomes negative and for large $+^{ve}$ values of $x$ is again a large negative number.

Fig. 46 shews the 12 points obtained by plotting numbers in the first column horizontally against numbers in the third column vertically. The regularity of the variation in the value of the expression is at once apparent; but, before a curve can be drawn, other points must be obtained for the region near the bend. These other points also serve as a check on the continuity of the graph.

| Values of $x$. | $7·5 - x$. | $x(7·5 - x)$. |
|---|---|---|
| −0·5 | 8 | −4·0 |
| 0 | 7·5 | 0 |
| 0·5 | 7·0 | 3·5 |
| 1·0 | 6·5 | 6·5 |
| 1·5 | 6·0 | 9·0 |
| 2·0 | 5·5 | 11·0 |
| 2·5 | 5·0 | 12·5 |
| 3·0 | 4·5 | 13·5 |
| (1) | | |
| 3·2 | 4·3 | 13·76 |
| 3·5 | 4·0 | 14·0 |
| 3·6 | 3·9 | 14·04 |
| 3·7 | 3·8 | 14·06 |
| 3·8 | 3·7 | 14·06 |
| 3·9 | 3·6 | 14·04 |
| 4·0 | 3·5 | 14·0 |
| (2) | | |
| 4·5 | 3·0 | 13·5 |

and the first and second columns are now interchanged if $x$ be increased by 0·5 each time.

That is to say, the first column reading downwards from line (2) is the same as the second column reading upwards from line (1); while the second column reading downwards from line (2) is the same as the first column reading upwards from line (1).

*N.B.*—An observation of this kind clearly reduces the amount of calculation otherwise necessary.

Points are plotted as before, using a larger scale, *i.e.* magnifying a portion of the last figure.

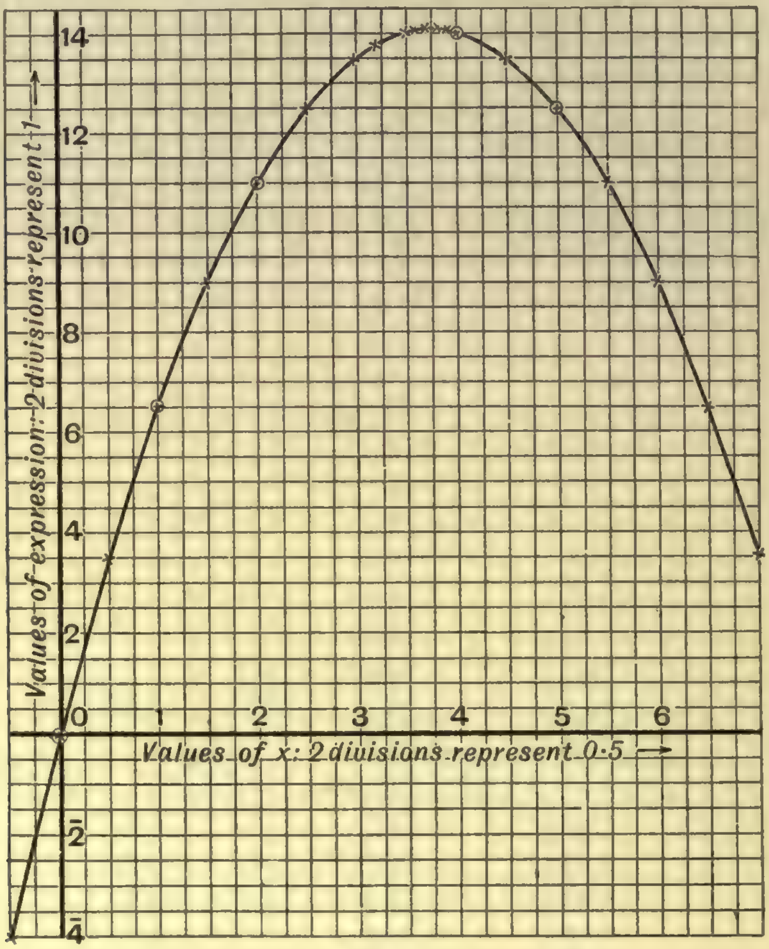


FIG. 47.

The perfect regularity of the curve formed by the plotted points is now quite clear (Fig. 47), and we may draw a graph to pass through all these points and be certain that if any value of $x$, not in the table, be selected and the value of the expression found, the point representing this value will also lie on the curve. From the curve we learn

(1) That the expression $x(7{\cdot}5-x)$ reaches a maximum value of about 14·06, and that it has this value when $x$ lies between 3·7 and 3·8. The exact maximum and the value of $x$ giving this maximum could be found by magnifying the part of the curve between, say, $x=3{\cdot}6$ and $x=3{\cdot}9$. To 5 significant figures the maximum is 14·063, and $x$ is then 3·75.

(2) As $x$ increases from a large negative number, the expression increases, rapidly at first, then more slowly as $x$ approaches the value 3·75. As $x$ passes 3·75, the expression begins to decrease slowly, and as $x$ continues to increase, the expression decreases more and more rapidly and without limit.

(3) The expression $x(7·5-x)$ clearly cuts the axis of $x$, that is to say, is equal to 0, where $x=0$ and where $x=7·5$ (see Fig. 46).

## EXERCISES XXIX.

**1.** The table shews the value of the imports into the United Kingdom for the last 10 years. Exhibit this graphically.

| Year - - - | 1902. | 1903. | 1904. | 1905. | 1906. |
|---|---|---|---|---|---|
| Value of Imports in millions of £'s | 528 | 543 | 551 | 565 | 608 |
| Year - - - | 1907. | 1908. | 1909. | 1910. | 1911. |
| Value of Imports in millions of £'s | 646 | 593 | 625 | 678 | 681 |

**2.** The table gives the population of the United Kingdom in millions at each census from 1821 to 1911 :

| Year - - - | 1821. | 1831. | 1841. | 1851. | 1861. |
|---|---|---|---|---|---|
| England and Wales | 12 | 13·9 | 15·9 | 17·9 | 20·1 |
| Scotland - - | 2·10 | 2·36 | 2·60 | 2·89 | 3·06 |
| Ireland - - - | 6·80 | 7·77 | 8·18 | 6·55 | 5·80 |
| Year - - - | 1871. | 1881. | 1891. | 1901. | 1911. |
| England and Wales | 22·7 | 26·0 | 29·0 | 32·5 | 36·1 |
| Scotland - - | 3·36 | 3·74 | 4·03 | 4·47 | 4·76 |
| Ireland - - - | 5·41 | 5·17 | 4·70 | 4·46 | 4·38 |

Determine, approximately, the population for each division of the kingdom in 1857 and in 1897. About what year was the population of England and Wales 30 millions? When was the population of Scotland about 3·5 millions? When was the population of Ireland about 5 millions? About what time did the population of Ireland begin to decrease? Is it decreasing more or less rapidly now? How does the rate of increase of population in England and Scotland at the present time compare with its rate in 1851?

**3.** In 1908 an income-tax of one shilling in the £1 was subject to abatements as shewn:

| INCOMES. | | Abatement. |
|---|---|---|
| Exceeding. | Not Exceeding. | |
| — | 160 | No tax |
| 160 | 400 | No tax on first £160 |
| 400 | 500 | „ £150 |
| 500 | 600 | „ £120 |
| 600 | 700 | „ £70 |
| 700 | — | No abatement |

Draw a graph to shew the tax on all incomes from £150 to £900 per annum.

*Note.*—Complete the table by adding columns headed 'taxable income,' 'total tax at 1s. per £1.'

**4.** The following table gives what is called 'the expectation of life' at various ages. Illustrate graphically.

| Age in Years - - | 10 | 15 | 20 | 25 | 30 | 35 | 40 | 45 | 50 |
|---|---|---|---|---|---|---|---|---|---|
| Expectation of Life | 49·6 | 45·2 | 41·0 | 37·0 | 33·1 | 29·2 | 25·6 | 22·2 | 18·9 |

At what age can a man expect to live 20 more years?

What is the expectation of life at 27, at 42?

**5.** From the given table plot curves shewing (i) the hour of sunrise, (ii) the hour of sunset, (iii) the duration of the day from sunrise to sunset, for the given period. Deduce the dates of (i) the latest sunrise, (ii) the earliest sunset, (iii) the shortest day.

| Date. | | Sunrise. | | | Sunset. | | |
|---|---|---|---|---|---|---|---|
| | | hrs. | min. | | hrs. | min. | |
| Nov. | 26 | 7 | 37 | a.m. | 3 | 57 | p.m. |
| Dec. | 3 | 7 | 48 | „ | 3 | 51 | „ |
| „ | 10 | 7 | 58 | „ | 3 | 47 | „ |
| „ | 17 | 8 | 4 | „ | 3 | 47 | „ |
| „ | 24 | 8 | 8 | „ | 3 | 50 | „ |
| „ | 31 | 8 | 9 | „ | 3 | 56 | „ |
| Jan. | 7 | 8 | 7 | „ | 4 | 4 | „ |
| „ | 14 | 8 | 4 | „ | 4 | 14 | „ |
| „ | 21 | 7 | 56 | „ | 4 | 26 | „ |

**6.** Make a list of the squares of the numbers 0, 1, 2, 3, 4 and 5, and draw a graph to shew the squares of all numbers between 0 and 5. From your graph determine the squares of 2·5, 3·2 and 1·7. What are the numbers whose squares are most nearly 10 and 20?

**7.** The cost of fuel per week for engines of various horse-powers are given in the table:

| Cost of fuel - - - | 5/- | 9/6 | 19/- | 31/6 | 36/6 |
|---|---|---|---|---|---|
| Horse-power of engine | 10 | 20 | 40 | 80 | 100 |

Exhibit the table graphically, and estimate the probable cost of fuel for engines of 25 and 90 horse-power.

Make a new table shewing the *cost of fuel per week per horse-power* for engines of various powers, and illustrate graphically.

*Note.*—For an engine of 10 H.P. the fuel costs 5*s.* per week; therefore the cost of fuel per week per horse-power is 6*d.* in this case. For the engine of 40 H.P. the cost is 18*s.* per week; therefore in this case the cost per week per horse-power is 5·4*d.*

**8.** For a certain machine, the table shews the effort P kilograms required to overcome a resistance W kilograms:

| W | 50 | 100 | 200 | 300 | 400 | 500 | 600 |
|---|---|---|---|---|---|---|---|
| P | 104 | 122 | 152 | 185 | 216 | 248 | 278 |

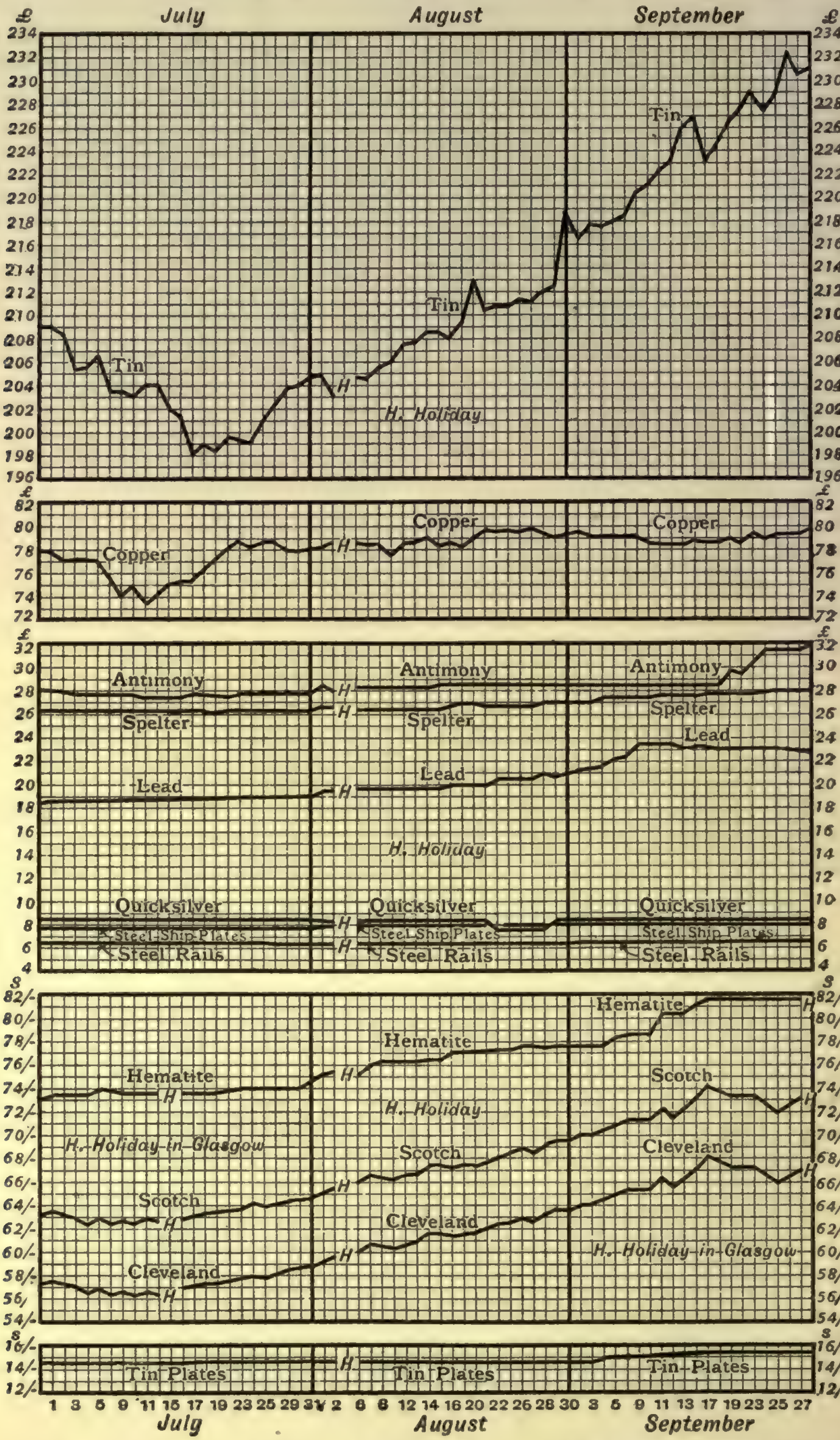


FIG. 48.—Variations in the prices of certain metals for three months of 1912. (By permission of the Editor of *Engineering*.)

Exhibit the table graphically.

As the table gives the efforts as determined experimentally, there are almost sure to be errors of experiment. Draw a graph through the mean of the points, and say what is the probable error in the measured value of P when W is 100, when W is 500 and also when W is 600. What effort is required for a resistance of 350 kilos.?

**9.** Fig. 48 shews graphically three months' fluctuations in the prices of certain metals.

What is the general trend, during these three months, of the prices of the several metals?

How do you account for the close resemblance between the curves labelled Haematite, Scotch, Cleveland?

What peculiarity do you notice about the curves for quicksilver, steel ship plates, steel rails; and how do you account for it?

For which metal has the variation in price during the three months been greatest?

Suppose that the price of tin had increased quite uniformly from the actual price at the end of July, how many pounds per ton per month should this uniform increase be for the price of tin to be what it actually was at the end of September?

Answer corresponding questions in the cases of the other metals.

**10.** The formula $y=2+\frac{x}{3}$ expresses the relation between the two variable quantities $x$ and $y$. Calculate the value of $y$ for each of the following values of $x$: 0, 1, 2, 3, 4, 5, 6. On squared paper draw the graph of the given expression, plotting values of $x$ horizontally and values of $y$ vertically. From the graph, find the value of $x$ when $y=3{\cdot}5$. (U.L.C.I.)

**11.** The following results shew the length of an elastic cord stretched by various loads. Plot the results, and determine the length of the cord unstretched. Also find what load will make the length 15 inches.

| | | | | | |
|---|---|---|---|---|---|
| Load in ounces: | 4, | 6, | 9, | 14, | 17. |
| Length in inches: | 13·1, | 13·5, | 14·1, | 15·1, | 15·7. |

(M.C.U.)

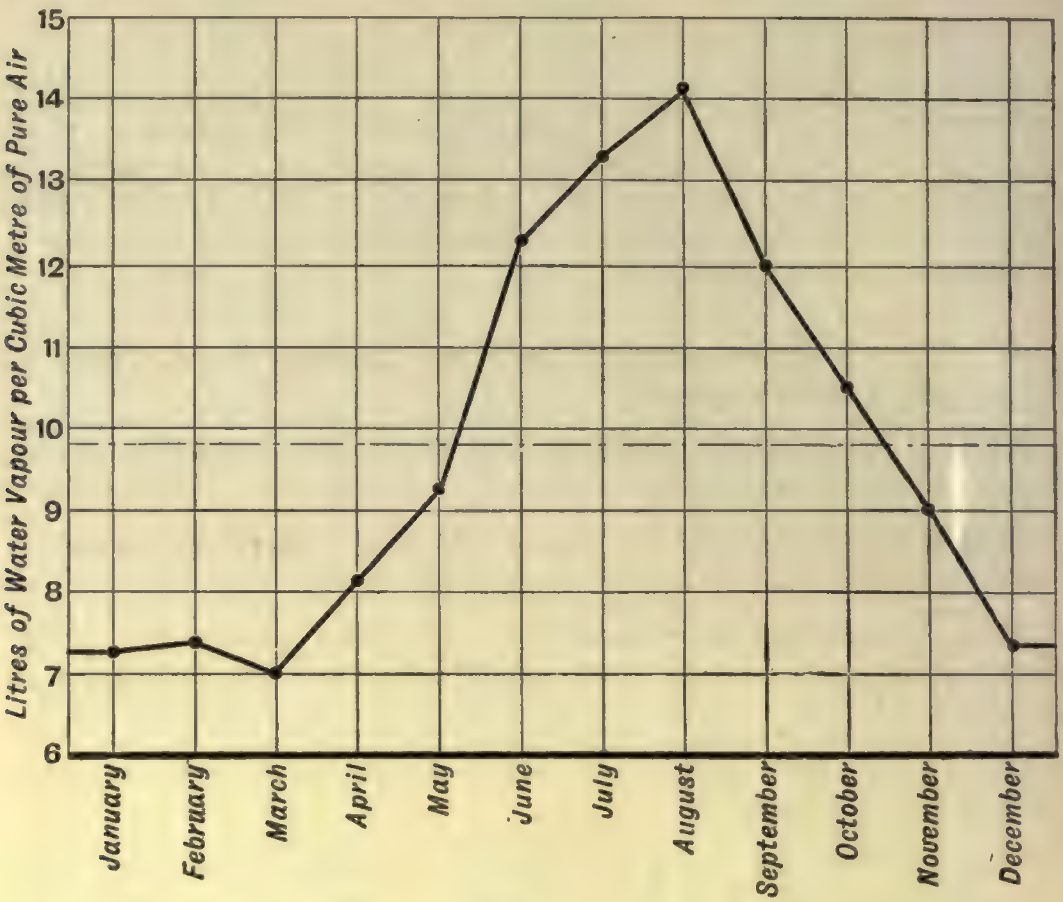


FIG. 49.

**12.** Fig. 49, from the Observatory Department of the National Physical Laboratory, shews the mean humidity over three years for each month of the year. How many litres of water vapour on the average were there in each cubic metre of pure air in February, June and October?

In which month was the humidity (i) greatest, (ii) least?

What was the approximate monthly rate of decrease of humidity from August to December?

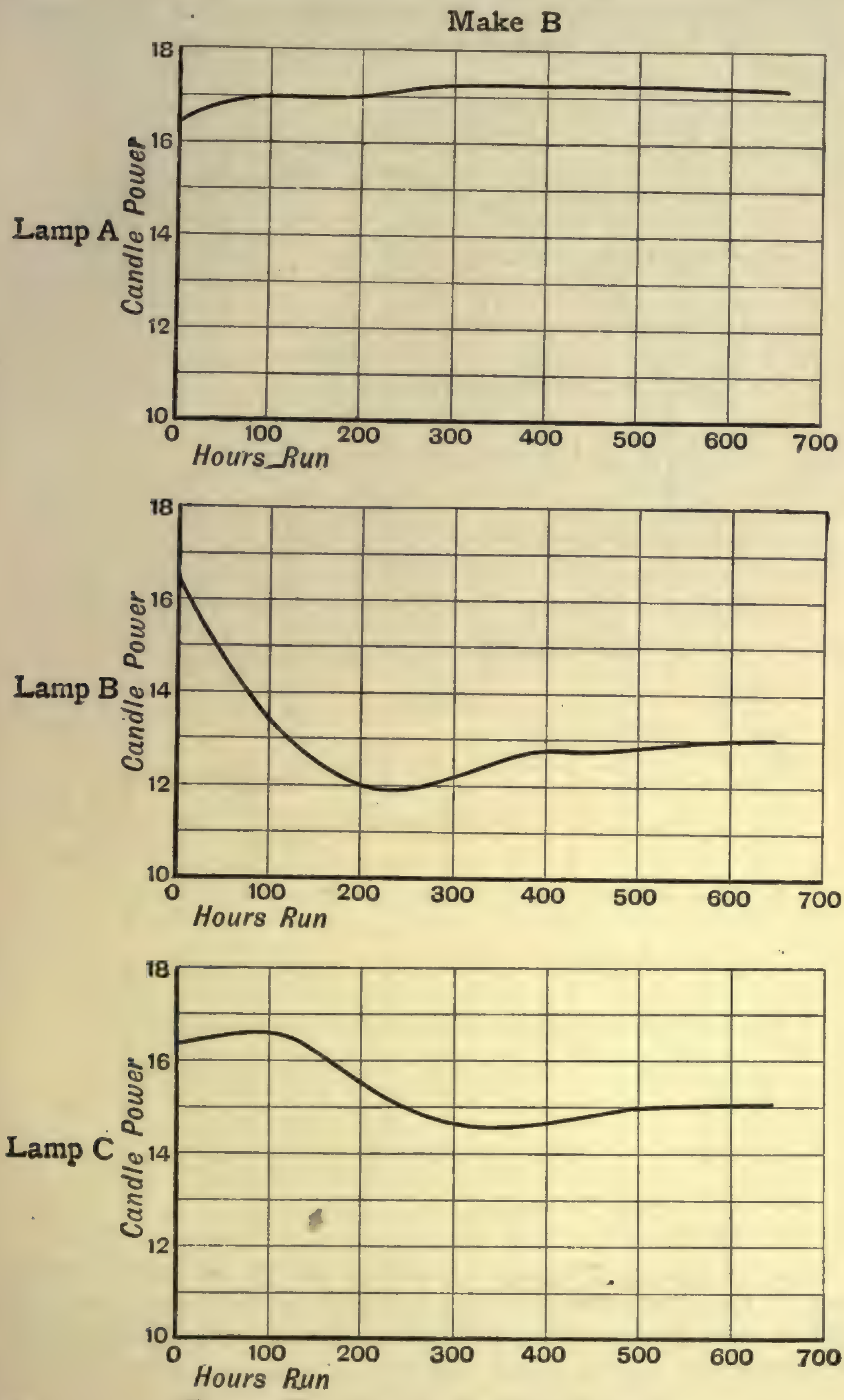
**13.** Fig. 50 shews the candle-powers of three lamps after varying numbers of hours' use.

Which lamp burnt most steadily, and what was its average candle-power?

During what period did the candle-power of lamp B decrease most rapidly?

What is the average candle-power of lamp C per 600 hours' run?

In what general respect do lamps B and C apparently differ from lamp A?



Make B

Lamp A

Lamp B

Lamp C

FIG. 50—Illustrating the life of electric lamps.

**14.** Figs. 51, 52 shew a load-extension diagram for copper wire. Fig. 52 is a magnified view of the straight portion of Fig. 51.

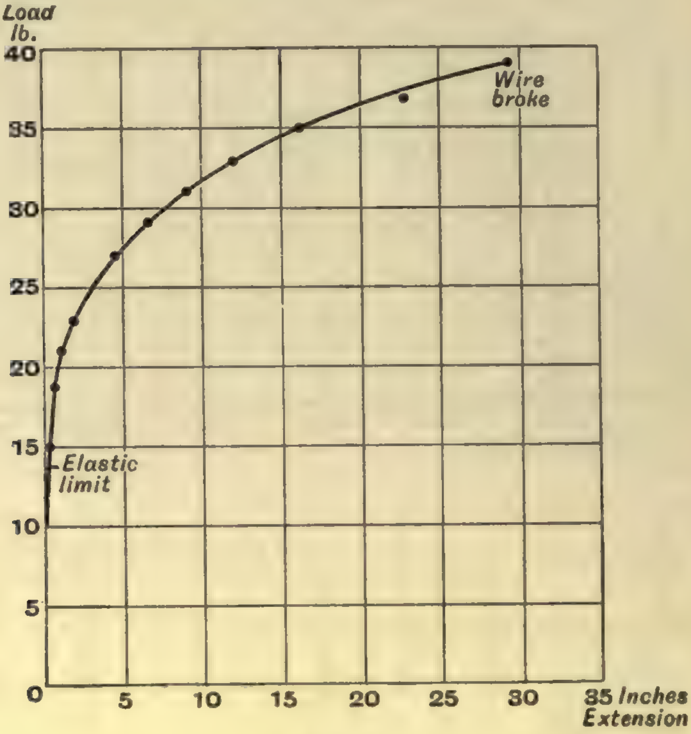


FIG. 51.

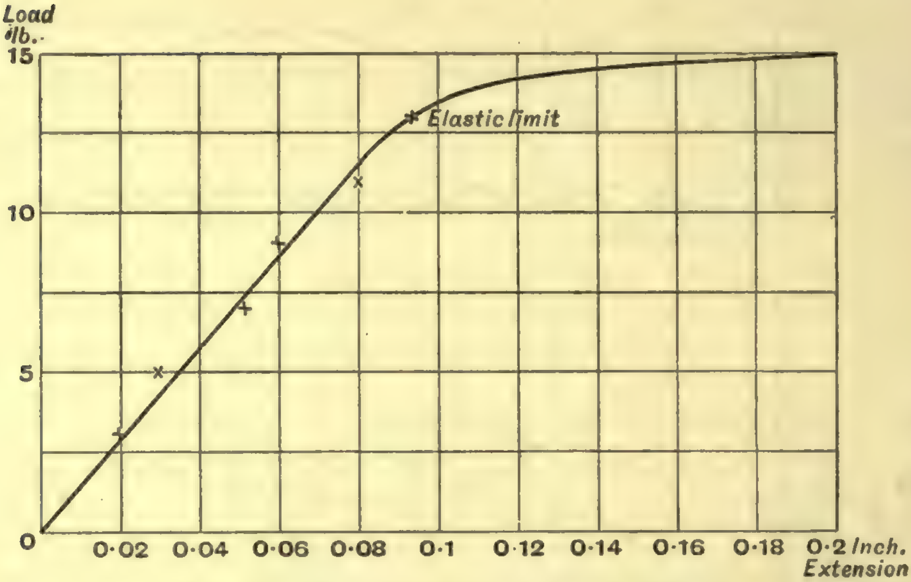


FIG. 52.—Magnified view of part of Fig. 51.

What is the approximate extension per lb. of load? If the area of the wire is 0·0011 sq. in., what is the stress (*i.e.* the load in tons per sq. inch) (*a*) at the elastic limit, (*b*) when the wire broke.

**15.** Fig. 53 shews the voltage required at different current densities for two carbon brushes A and B.

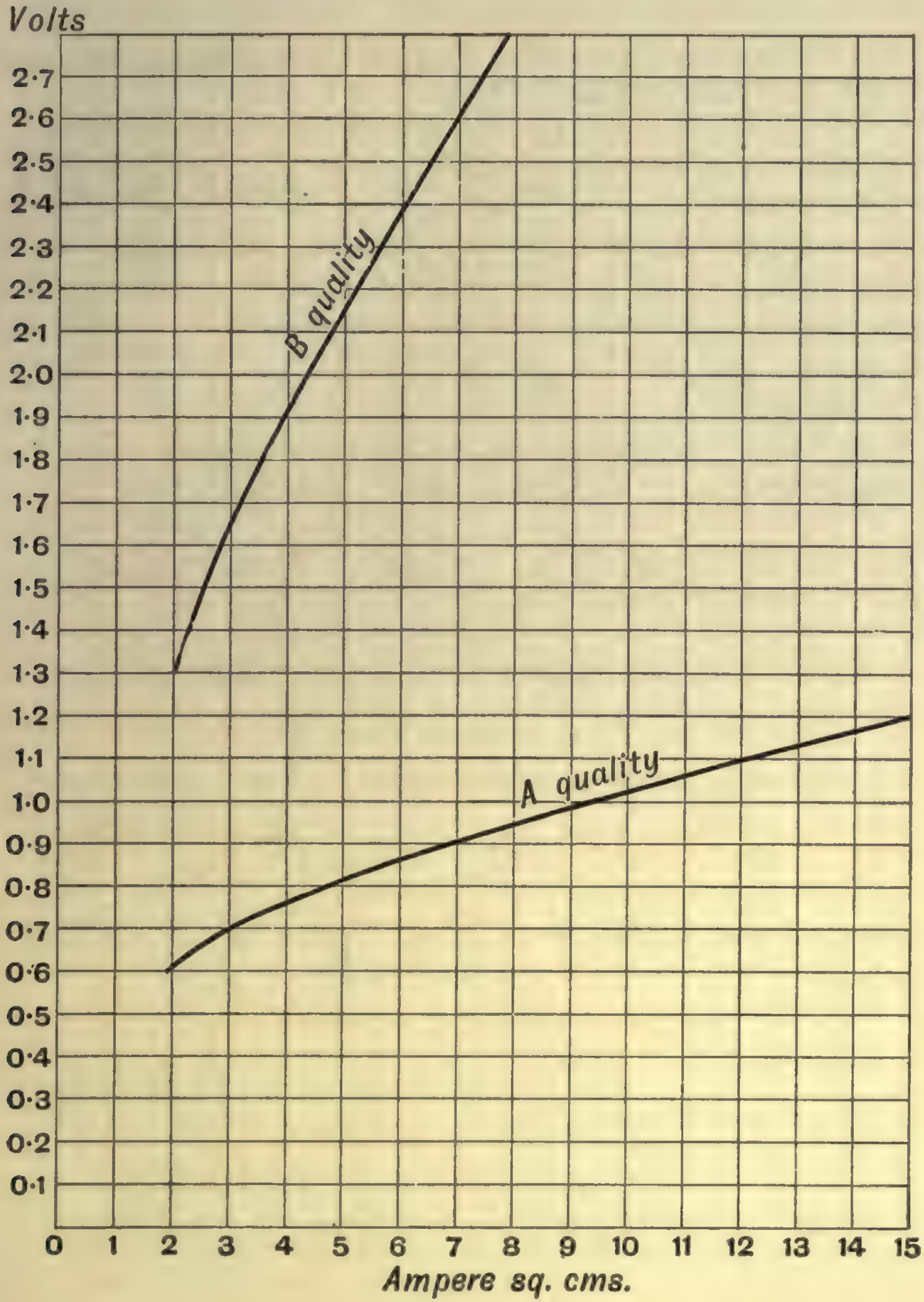


FIG. 53.

Find the average voltage required for each brush for current densities ranging from 2 to 7.

What are the voltages required for brush A at 5 ampères per sq. cm., 10 ampères per sq. cm., 15 ampères per sq. cm.?

If the safe current density is 9 ampères per sq. cm. for A quality and 4 ampères per sq. cm. for B quality, what area of brush is required to collect 100 ampères in each case, and what are the total numbers of watts lost? (1 watt = 1 volt × 1 ampère.)

**16.** The melting point of a mixture of antimony and copper is shewn in the following table:

| Percentage of copper in the mixture | 25 | 30 | 35 | 40 | 45 | 50 | 55 | 60 | 65 |
|---|---|---|---|---|---|---|---|---|---|
| Melting point - - | 480 | 530 | 565 | 598 | 627 | 650 | 660 | 650 | 630 |

Illustrate this graphically, and thence determine the melting point of a mixture containing 33 per cent. of copper. (M.C.U.)

**17.** R and V denote respectively the resistance in lb. per ton and the speed in miles per hour of a train:

| R | 8 | 8·6 | 10·3 | 13·3 | 17·4 | 22·6 | 29·1 |
|---|---|---|---|---|---|---|---|
| V | 0 | 10 | 20 | 30 | 40 | 50 | 60 |

Draw a curve shewing how the speed increases with the resistance.

# CHAPTER VII.

## FACTORS. H.C.F., L.C.M. FRACTIONS.

**Factors.**—A **factor** of a number is another number which exactly divides it : *e.g.* 2 is a factor of 12, 6 is a factor of 12, $a$ and $b$ are factors of $ab$, $x$ is a factor of $x(y+z)$, etc.

A **prime number**, or **prime**, is a number which has no factors except itself and unity : *e.g.* 2, 3, 5, 13, 137, etc., are prime numbers.

A **prime factor** of a number is a prime which exactly divides the given number : *e.g.* 2 is a prime factor of 12, and so is 3 ; but 6, although a factor, is not a prime factor of 12.

Ex. 1. *Express* 108 *as the product of prime factors.*

The process is as follows : Find the smallest prime which divides 108, viz. 2, and

$$108=2\times 54.$$

The smallest prime factor of 54 is 2, and

$$54=2\times 27\,;$$

$$\therefore\ 108=2\times 2\times 27.$$

The smallest prime factor of $27=3$, and

$$27=3\times 9=3\times 3\times 3\,;$$

$$\therefore\ 108=2\times 2\times 3\times 3\times 3.$$

Now, as 2 and 3 are prime numbers, 108 is expressed as the product of prime factors.

Evidently 108 can be written as $2^2\times 3^3$.

Ex. 2. *Express* 504 *as the product of prime factors.*

$$\begin{aligned}504=2\times 252=2\times 2\times 126&=2\times 2\times 2\times 63\\&=2^3\times 3\times 21\\&=2^3\times 3^2\times 7.\end{aligned}$$

A **common factor** of two or more numbers is a number which exactly divides each: *e.g.*

2 is a common factor of 12 and 18;
3 is a common factor of 12, 18 and 9;
6 is a common factor of 12, 18 and 30.

Again, $a$ is a common factor of $ax$ and $ay$;
$z$ is a common factor of $zp$, $zq$, $z(c+d)$ and $kz(a+b)$;
$(a+b)$ is a common factor of $(a+b)^2$, $c(a+b)$ and $(p+q)(a+b)$.

Ex. 3. *Find the common factors of* $(p+q)^3$, $(c+d)(p+q)^4$ *and* $(c+d)^2z(p+q)^2$.

By inspection $(p+q)^2$ is the only common factor.

Ex. 4. *Find the common factors of*

$$abc(y+z)^2,\ ad^4(y+z)(x+y),\ a^3bd(y+z)^3(a+b).$$

By inspection $a$ and $(y+z)$ are the only factors common to each expression.

## Highest Common Factor.

[*This paragraph is not of practical importance, and need only be read by students taking examinations in which the use of logarithms is not allowed or which are set by authorities whose syllabus includes* H.C.F.]

The **highest common factor**, or **H.C.F.**, as it is shortly written, of two or more numbers is the largest number which exactly divides each. The best method to employ is to express each number in its prime factors, and then, by inspection, the H.C.F. can easily be written down.

Ex. 1. *Find the* H.C.F. *of* 240 *and* 360.

$$240=2\times120=2^2\times60=2^3\times30=2^4\times15=2^4\times3\times5.$$

$$360=2\times180=2^2\times90=2^3\times45=2^3\times3\times15=2^3\times3^2\times5.$$

Now $2^3$ is the highest power of 2 which exactly divides both numbers, 3 is the highest power of 3 which exactly divides them and 5 is the highest power of 5. Hence the largest number which exactly divides 240 and 360 must contain $2^3$ and 3 and 5, and is therefore $2^3\times3\times5$, *i.e.* $8\times3\times5$ or 120; *i.e.* 120 is the H.C.F. of 240 and 360.

Ex. 2. *Find the* H.C.F. *of* 550, 780 *and* 70.

$$550=2\times 275=2\times 5\times 55=2\times 5^2\times 11.$$

$$780=2\times 390=2^2\times 195=2^2\times 3\times 65=2^2\times 3\times 5\times 13.$$

$$70=2\times 35=2\times 5\times 7.$$

The H.C.F. is $2\times 5$, *i.e.* 10 : for 2 and 5 are the highest powers of these numbers which exactly divide each of the three given numbers, so that the H.C.F. must contain 2 and 5. But 13 and 3 only divide 780, 7 only divides 70 and 11 only divides 550 ; and consequently the H.C.F. cannot contain 3, or 7, or 11, or 13.

## EXERCISES XXX.

**1.** Express 24, 75, 115, 225, 764 as products of prime numbers.

**2.** Write down the factors of $abc$, $2ax$, $6p(r+s)$.

**3.** Write down all the factors of $\left(\frac{v+u}{2}\right)t$, $\frac{n}{2}\{2a+(n-1)d\}$, $\frac{(v-u)(v+u)}{2f}$, $\mathrm{P}\left(1+\frac{r}{100}\right)^2$, $(a+b)(c+d)$, $a+b(c+d)$.

**4.** Write down the common factors of

(i) $ax$, $ay$, $az$.

(ii) $a^2bx$, $ab(x+y)$, $acp^2b^2$.

(iii) $2l(p+q)^2$, $3m(p+q)$, $6(r+s)(p+q)$.

(iv) $10(y+z)^2(a+b)$, $5(y+z)^3(y+x)$, $40p(y+z)^2(a+b)$.

**5.** Find the H.C.F. of

(i) 12, 15, 30, 72.

(ii) 252, 308, 420.

(iii) 60, 105, 195, 210.

(iv) 936, 9450, 1764.

**6.** Find the common factors of

(i) $ab$, $3ac$, $2ad$.

(ii) $6p(q+r)$, $18p^2s$, $15pz$.

(iii) $12x^2y$, $30x^3(z+w)$, $24x^4(y+p+c)$.

(iv) $12a^3bc$, $15ab^2c^3$, $30a^4bc^2$.

**The Lowest Common Multiple of two or more numbers is the smallest number which exactly contains them.**

Ex. 1. *Find the lowest common multiple* (L.C.M.) *of* 24 *and* 40.

Expressing each number in its prime factors,

$$24=2\times 12=2^2\times 6=2^3\times 3.$$

$$40=2\times 20=2^2\times 10=2^3\times 5.$$

Now, the least number into which 24 and 40 divide exactly must contain $2^3$ and 3 to be divisible by 24, and $2^3$ and 5 to be divisible by 40.

Hence it must contain $2^3$, 3 and 5.

Consequently the L.C.M. is $2^3\times 3\times 5$ or 120.

Ex. 2. *Find the* L.C.M. *of* 54a, 120ab *and* $180a^2d$.

$$54a=2\times 27a=2\times 3^3\times a.$$

$$120ab=2\times 60ab=2^2\times 30ab=2^3\times 3\times 5\times a\times b.$$

$$\begin{aligned}180a^2d=2\times 90a^2d=2^2\times 45a^2d&=2^2\times 3\times 15a^2d\\&=2^2\times 3^2\times 5\times a^2\times d.\end{aligned}$$

To obtain the L.C.M., take the highest powers of 2, 3 and 5, a, b and d, and multiply them together.

$$\begin{aligned}\therefore\ \text{L.C.M.}&=2^3\times 3^2\times 5\times a^2\times b\times d\\&=360a^2bd.\end{aligned}$$

The sum of the two fractions $\frac{1}{2}$ and $\frac{1}{3}$ is the same as the sum of $\frac{3}{6}$ and $\frac{2}{6}$, viz. $\frac{5}{6}$. The process of expressing fractions in such a form that their sum or difference may be written down by inspection involves finding the L.C.M. of certain numbers. Thus, 6 is the L.C.M. of 2 and 3, and this number must be used in evaluating $\frac{1}{2}+\frac{1}{3}$.

Ex. 3. *Find* $\frac{2}{3}+\frac{1}{5}$.

The L.C.M. of 3 and 5 is 15; and clearly $\frac{2}{3}=\frac{10}{15}$ and $\frac{1}{5}=\frac{3}{15}$.

$$\begin{aligned}\therefore\ \frac{2}{3}+\frac{1}{5}&=\frac{10}{15}+\frac{3}{15}\\&=\frac{13}{15}.\end{aligned}$$

Ex. 4. *Express as a single fraction* $\frac{1}{a^2}-\frac{1}{ab}$.

The L.C.M. of $a^2$ and $ab$ is $a^2b$.

$$\frac{1}{a^2}=\frac{b}{a^2b},$$

$$\frac{1}{ab}=\frac{a}{a^2b}.$$

$$\therefore\ \frac{1}{a^2}-\frac{1}{ab}=\frac{b}{a^2b}-\frac{a}{a^2b}$$

$$=\frac{b-a}{a^2b}.$$

## EXERCISES XXXI.

**1.** Find the L.C.M. of

(i) 4, 6, 15, 50. (ii) 75, 100, 120.

(iii) $6a$, $10b$. (iv) $ab$, $ac$.

(v) $2a^2$, $6a$, $12ab$. (vi) $4a^2b$, $6ax$, $9bc$.

(vii) $2a(x+y)$, $4bc(p+q)$, $3ac(x+y)^2$.

**2.** Find the H.C.F. and L.C.M. of

(i) $a^2b$, $b^2ac(d+x)$, $(d+x)^2ab$.

(ii) $ae^3(l+m)$, $ae(l+m)^2$, $a^2be(l+m)(m+n)$.

**3.** Find the smallest number which, when divided by 12, 18, 27 and 36, gives in each case a remainder 4.

**4.** Express as single fractions :

(i) $\frac{1}{3}+\frac{1}{4}+\frac{1}{6}$. (ii) $\frac{1}{2}+\frac{1}{8}-\frac{1}{3}$. (iii) $\frac{1}{a}-\frac{1}{b}$.

(iv) $\frac{1}{x}+\frac{1}{y}-\frac{1}{z}$. (v) $\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}+\frac{1}{R_4}$. (vi) $\frac{1}{2a}+\frac{1}{3a}-\frac{1}{4a}$.

(vii) $\frac{x}{4}+\frac{5x}{12}$. (viii) $\frac{2}{yz}+\frac{3}{5yz}$. (ix) $\frac{1}{3x}+\frac{1}{2x}-\frac{1}{6x}$.

(x) $\frac{1}{2a}+\frac{1}{3b}-\frac{1}{4c}$. (xi) $\frac{a}{b}+\frac{a}{3b}-\frac{c}{2b}$. (xii) $\frac{6}{a^2bc}-\frac{4}{a^2b}+\frac{3}{ac^2}$.

**Fractions.**—A fraction in its first and literal meaning is a part of a whole. Thus, in scale drawing, the *representative fraction* of the scale is given.

| | |
|---|---|
| So, 2*s*. is a fraction of £1, | viz. one-tenth. |
| 4 oz. is a fraction of 1 lb., | viz. one-quarter. |
| 2 feet is a fraction of 1 yd., | viz. two-thirds. |
| 1 mm. is a fraction of 1 cm., | viz. one-hundredth. |
| 1 pint is a fraction of 1 gallon, | viz. one-eighth. |

Now, though the literal meaning of a fraction is a part of a whole, it is convenient to apply the term fraction to any expression of the form $\frac{a}{b}$, where $a$ and $b$ are any numbers or expressions whatever. Hence, to evaluate a fraction arithmetically, it is only necessary to perform a division sum.

Thus, $\frac{10}{5}$ is a fraction; its value is 2.

$\frac{4}{3}$ is a fraction; its value is 1·333 to 4 significant figures.

$\frac{23}{79}$ is a fraction; its value is 0·2911.

$$\begin{array}{r|l} & 2911 \\ \hline 79 & 230 \\ & \;\;720 \\ & \quad\;\; 90 \\ & \quad\;\; 11 \end{array}$$

$\frac{2\frac{4}{5}}{4}$ is a fraction, and equals $\frac{2{\cdot}8}{4}$ or 0·7.

$\frac{8}{3\frac{1}{5}}$ is a fraction, and equals $\frac{8}{3{\cdot}2}$ or 2·5.

$\frac{\frac{3}{5}}{1\frac{1}{2}}$ is a fraction, and equals $\frac{0{\cdot}6}{1{\cdot}5}$ or 0·4.

So also $\frac{a/x}{b/y}$ is a fraction. The method of simplifying such a fraction is given on p. 125.

In any fraction $\frac{a}{b}$, $a$ is called the **numerator** and $b$ is called the **denominator**.

We have assumed, and it can be shown, that **any fraction is unaltered in value if its numerator and denominator are each multiplied by the same number.**

Thus,
$$\frac{1}{2}=\frac{2}{4}=\frac{3}{6}=\frac{5}{10}=\frac{9}{18}, \text{ etc.}$$
$$\frac{3\frac{1}{3}}{4\frac{1}{2}}=\frac{3\frac{1}{3}\times 6}{4\frac{1}{2}\times 6}=\frac{20}{27}.$$
$$\frac{a}{b}=\frac{ax}{bx}.$$

**Any fraction is unaltered in value if its numerator and denominator are each divided by the same number.**

Thus,
$$\frac{15}{20}=\frac{3}{4}.$$
$$\frac{a}{b}=\frac{a/y}{b/y}.$$

Use is made of this principle in **cancelling**, or simplifying complex fractions.

Ex. *Simplify the expression* $\dfrac{a(a+b)^2(c-d)^2(a-b)}{a^2b(c+d)(a-b)^2(a+b)}$.

Dividing top and bottom by $a(a+b)(a-b)$, we have

$$\text{the expression}=\frac{(a+b)(c-d)^2}{ab(c+d)(a-b)}.$$

## EXERCISES XXXII.

Find the value of

1. $\dfrac{1\frac{1}{2}}{2}$. 2. $\dfrac{24}{2\frac{1}{4}}$. 3. $\dfrac{5\frac{3}{5}}{4}$. 4. $\dfrac{9\frac{1}{7}}{4\frac{4}{5}}$.

5. $\dfrac{\frac{2}{3}}{\frac{3}{4}}$. 6. $\dfrac{\frac{5}{7}}{6\frac{7}{8}}$. 7. $\dfrac{8\frac{2}{3}}{5\frac{4}{7}}$. 8. $\dfrac{3\frac{1}{17}}{4\frac{1}{51}}$.

Simplify

9. $\dfrac{48}{64}$. 10. $\dfrac{15}{75}$. 11. $\dfrac{13}{39}$. 12. $\dfrac{4\times 12}{8\times 36}$.

13. $\dfrac{4\times 7\times 51}{49\times 34}$. 14. $\dfrac{1\frac{1}{2}\times\frac{3}{5}}{9\times\frac{2}{5}}$. 15. $\dfrac{4\frac{1}{3}}{65}$.

16. $\dfrac{abc}{a^2b}$. 17. $\dfrac{c(x+y)}{d(x+y)^2}$. 18. $\dfrac{(x-y)(p+q)}{(p+q)^2(x+y)}$.

19. $\dfrac{a+ab}{a(a+b)}$. 20. $\dfrac{x-xy}{x(x-y)}$. 21. $\dfrac{x^2+xy}{y^2+xy}$.

**Manipulation of Fractions.**—Any fraction in which the numerator is greater than the denominator can be expressed as a whole number, plus a fraction less than unity. Thus, $\frac{4}{3}$ means four-thirds, *i.e.* three-thirds plus one-third, *i.e.* $1+\frac{1}{3}$ or $1\frac{1}{3}$.

Again, $$\frac{103}{100}=1+\frac{3}{100}=1\tfrac{3}{100},$$

$$\frac{13}{5}=2\tfrac{3}{5}.$$

$$\frac{ab+c}{a}=\frac{ab}{a}+\frac{c}{a}.$$

Now, a fraction is unaltered if numerator and denominator are each divided by the same number.

$$\therefore \quad \frac{ab}{a}=\frac{b}{1}=b.$$

Hence $$\frac{ab+c}{a}=b+\frac{c}{a}.$$

Similarly, $$\frac{x^3y+x^2}{x^3}=\frac{x^3y}{x^3}+\frac{x^2}{x^3}$$

$$=y+\frac{1}{x}.$$

Conversely, any number can be expressed as a fraction. Thus, 2 is four-halves, *i.e.* $\frac{4}{2}$, or ten-fifths, *i.e.* $\frac{10}{5}$.

Again, $1\frac{13}{16}$ is sixteen-sixteenths plus thirteen-sixteenths, *i.e.* 29 sixteenths or $\frac{29}{16}$.

$$p+\frac{r}{q}=\frac{pq}{q}+\frac{r}{q}=\frac{pq+r}{q}.$$

$$\frac{1}{x}-\frac{1}{y}=\frac{y}{xy}-\frac{x}{xy}=\frac{y-x}{xy}.$$

Ex. 1. *Express* $\frac{2}{5}, \frac{4}{9}, \frac{11}{20}, \frac{37}{30}$ *with denominator* 180 *in each case.*

Clearly $$\frac{2}{5}=\frac{2\times 36}{5\times 36}=\frac{72}{180}, \qquad \frac{4}{9}=\frac{4\times 20}{9\times 20}=\frac{80}{180},$$

$$\frac{11}{20}=\frac{11\times 9}{20\times 9}=\frac{99}{180}, \qquad \frac{37}{30}=\frac{37\times 6}{30\times 6}=\frac{222}{180}.$$

Ex. 2. *Express* $\frac{2a}{b}, \frac{c}{d}, \frac{5p(c+d)}{4b(x-y)}$ *with denominator* $4bd(x-y)$ *in each case.*

$$\frac{2a}{b}=\frac{2a\times 4d(x-y)}{b\times 4d(x-y)}=\frac{8ad(x-y)}{4bd(x-y)},$$

$$\frac{c}{d}=\frac{c\times 4b(x-y)}{d\times 4b(x-y)}=\frac{4bc(x-y)}{4bd(x-y)},$$

$$\frac{5p(c+d)}{4b(x-y)}=\frac{5p(c+d)\times d}{4b(x-y)\times d}=\frac{5pd(c+d)}{4bd(x-y)}.$$

Ex. 3. *Express* $\frac{6}{21}$ *with its lowest denominator and* $\frac{28}{12}$ *in its simplest form.*

$\frac{6}{21}=\frac{3\times 2}{3\times 7}=\frac{2}{7}$ on dividing numerator and denominator by 3.

$$\frac{28}{12}=\frac{4\times 7}{4\times 3}=\frac{7}{3}=2\tfrac{1}{3}.$$

Ex. 4. *Simplify* $\frac{ab^2(c+d)(p+q)}{a^2b^3(p+q)}$.

We have $\frac{ab^2(c+d)(p+q)}{a^2b^3(p+q)}=\frac{ab^2(c+d)}{a^2b^3}$ ....................(i)

on dividing numerator and denominator by $(p+q)$,

$$=\frac{a(c+d)}{a^2b} \quad ..............................(ii)$$

on dividing numerator and denominator of (i) by $b^2$,

$$=\frac{c+d}{ab}$$

on dividing numerator and denominator of (ii) by $a$.

With a little practice, the intermediate steps (i), (ii) may be dispensed with; that is to say, the numerator and denominator of the original fraction may be divided by $ab^2(p+q)$ in one step.

## EXERCISES XXXIII.

1. Express $\frac{21}{18}, \frac{36}{27}, \frac{35}{105}, \frac{102}{100}, \frac{57}{38}$ with lowest denominators in each case, and give the value of each fraction to not more than four significant figures.

2. Express $\frac{2}{3}, \frac{3}{8}, \frac{5}{6}, \frac{11}{12}$ with denominator 24 in each case.

**3.** Express $\frac{x}{yz}$, $\frac{ab}{y^2(p+q)}$, $\frac{a+b}{yz^2(p+q)^2}$ with denominator $y^2z^2(p+q)^2$ in each case.

**4.** Write down the values of $\frac{2}{3} \times 15$, $12 \times \frac{4}{3}$, $\frac{2}{5} \times 20$, $\frac{3}{13} \times 65$, $20 \times \frac{4}{5}$, $\frac{a}{b} \times bc$, $\frac{x}{y} \times y(p+q)$, $\frac{a+b}{c^2x} \times cxz$.

**5.** Reduce to their lowest terms:

$$\frac{15}{25}, \ \frac{9}{27}, \ \frac{4xy}{2y^2}, \ \frac{6x^2y}{9xy}. \qquad \text{(U.L.C.I.)}$$

**6.** From a piece of wire 32 ft. long is cut a length of 1 ft. 3 ins. What fraction of the whole length has been removed, and what fraction of the original length remains? (U.L.C.I.)

**7.** On squared paper draw a rectangle 3″ by 2″ and divide it into $\frac{1}{2}$ in. squares.

What fractions of the whole area are 1 square, 3, 4 and 8 squares. Express each fraction in its lowest terms. (U.L.C.I.)

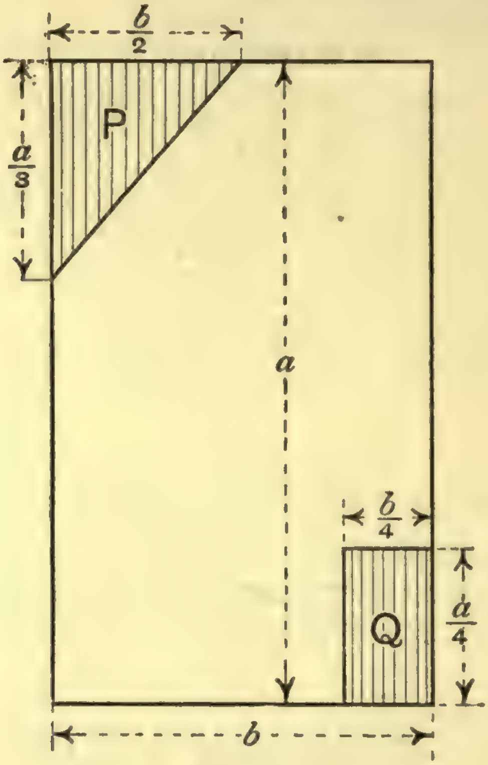


FIG. 54.

**8.** The given figure (Fig. 54) represents a rectangle, sides $a$ and $b$, with two portions P, Q shaded. What fractions of the total area are the areas P and Q, and what fraction of the original area remains when P and Q are removed? (U.L.C.I.)

**9.** Draw a rectangle ABCD and assume AB is $b$ in. long. Mark a point X in AB so that AX is $a$ in. Suppose AD to be divided into $m$ equal parts, and AB and AX to be graduated in inches. Divide ABCD into small rectangles by lines through the points of graduation of the sides.

Shew that the rectangle on base AX equals $\frac{a}{b}$ (rectangle ABCD).

Prove also that the rectangle on base AX $=\frac{am}{bm}$ (rectangle ABCD).

**10.** Simplify

(i) $\frac{x(x-2)(x+2)}{x^2(x+2)(x+3)}$. (ii) $\frac{6a^2b(p+q)}{72ab^2(p-q)}$.

(iii) $\frac{(x-1)(x-2)(x+1)(x-4)}{(x-4)(x-1)(2x+1)}$. (iv) $\frac{a^2(a+b)^2}{ax^4y(a+b)}$.

(v) $\frac{a(l-m)(l+m)^2(a+b)}{a^2(l+m)(a-b)}$. (vi) $\frac{12p(x-y)(a+b)}{142p^2qr(x-y+a+b)}$.

**Addition and Subtraction of Fractions.**—The truth of the following series of identities should be apparent:

$$4 \text{ apples} + 7 \text{ apples} = 11 \text{ apples};$$

$$5\ x\text{'s} + 3\ x\text{'s} = 8\ x\text{'s};$$

$$2 \text{ elevenths} + 5 \text{ elevenths} = 7 \text{ elevenths}$$

$$\text{or} \quad \tfrac{2}{11} + \tfrac{5}{11} = \tfrac{7}{11};$$

$$a \text{ sevenths} + b \text{ sevenths} = (a+b) \text{ sevenths}$$

$$\text{or} \quad \frac{a}{7} + \frac{b}{7} = \frac{a+b}{7}.$$

In the same way, we have

$$\frac{c}{10} - \frac{d}{10} = \frac{c-d}{10},$$

$$\frac{2}{x} + \frac{3}{x} = \frac{5}{x},$$

$$\frac{a}{x} + \frac{b}{x} = \frac{a+b}{x},$$

$$\frac{p}{y} - \frac{q}{y} + \frac{r}{y} = \frac{p-q+r}{y}.$$

To add or subtract fractions, all that is necessary is to express them with the same denominator. This "common denominator," as it is called, is conveniently the L.C.M. of the denominators of the original fractions.

Thus, $2\frac{1}{5}+3\frac{1}{4}=2\frac{4}{20}+3\frac{5}{20}$ (since L.C.M. of 5 and 4 is 20)

$$=5\tfrac{9}{20}.$$

$$3\tfrac{1}{4}+5\tfrac{13}{15}=8+\tfrac{1}{4}+\tfrac{13}{15}$$

$$* =8+\tfrac{15}{60}+\tfrac{52}{60}=8+\tfrac{67}{60}$$

$$=8+1+\tfrac{7}{60}=9\tfrac{7}{60}.$$

*Note.*—The step marked * is usually omitted.

$$3\tfrac{1}{4}-\tfrac{13}{15}=3+\frac{15-52}{60}=3+\frac{-37}{60}$$

$$=2+\frac{60-37}{60}=2\tfrac{23}{60}.$$

Ex. 1. *Simplify* $2\frac{1}{8}-13\frac{1}{4}+4\frac{1}{3}$.

$$2\tfrac{1}{8}-13\tfrac{1}{4}+4\tfrac{1}{3}=2-13+4+\frac{3-6+8}{24}$$

$$=-7+\frac{5}{24}=-6\tfrac{19}{24}.$$

Ex. 2. *Simplify* $\dfrac{p}{(q+r)x}+\dfrac{q}{x^2}-\dfrac{d}{x^3(q+r)}$.

L.C.M. of denominators $=x^3(q+r)$.

$$\text{expression}=\frac{px^2+qx(q+r)-d}{x^3(q+r)}.$$

**Multiplication and Division of Fractions.**—The following series of identities should be clear:

$$4\times\tfrac{3}{16}=\tfrac{12}{16}=\tfrac{3}{4};$$

$$c\times\frac{a}{b}=\frac{ca}{b};$$

$$\tfrac{15}{11}/5=\tfrac{3}{11}$$

(Note $\tfrac{15}{11}/5$ is the same as $\tfrac{1}{5}$ of $\tfrac{15}{11}$, *i.e.* $\tfrac{3}{11}$);

$$\frac{a}{b}\Big/c=\frac{a}{bc};$$

$\tfrac{7}{2}\times\tfrac{9}{22}$ means 7 halves of $\tfrac{9}{22}$,

*i.e.* 7 times $\tfrac{9}{44}$, *i.e.* $\tfrac{63}{44}$;

$$\tfrac{3}{4}\times\tfrac{17}{16}=\tfrac{51}{64};$$

$$\frac{a}{b}\times\frac{c}{d}=\frac{ac}{bd}.$$

The rule for multiplying two or more fractions together is: **Multiply the numerators together and divide by the product of the denominators.**

Again, $\tfrac{2}{3}/\tfrac{7}{5}$ is the same as $\dfrac{(\tfrac{2}{3}\times\tfrac{5}{7})}{\tfrac{7}{5}\times\tfrac{5}{7}}$, multiplying top and bottom of the fraction by $\tfrac{5}{7}$.

Hence $\tfrac{2}{3}/\tfrac{7}{5}=\tfrac{2}{3}\times\tfrac{5}{7}=\tfrac{10}{21}$.

Similarly, $\tfrac{4}{15}/\tfrac{3}{5}=\tfrac{4}{15}\times\tfrac{5}{3}=\tfrac{20}{45}=\tfrac{4}{9}$;

$$\frac{a}{b}\Big/\frac{c}{d}=\frac{a}{b}\times\frac{d}{c}=\frac{ad}{bc}.$$

The rule for dividing by a fraction is: **Invert the fraction and multiply.**

Note especially that $x\div\dfrac{1}{a}$ means $x\times a$, *i.e.* dividing by $\dfrac{1}{a}$ is the same as multiplying by $a$.

Remember that terms enclosed within a bracket are to be treated as a whole. Thus, $a\div b\times c$ and $a\div(b\times c)$ are not equivalent. The first means that $a$ is to be divided by $b$ and the quotient multiplied by $c$, *i.e.* $\dfrac{a}{b}\times c$, or $\dfrac{ac}{b}$. The second means that $a$ is to be divided by the product of $b$ and $c$, *i.e.* $\dfrac{a}{bc}$.

Ex. *Simplify* $\dfrac{2\frac{1}{2} \div 2\frac{1}{7} + \frac{5}{6}}{3\frac{7}{12} - 3\frac{1}{3} \text{ of } \frac{1}{60}}$. (M.C.U.)

$$\text{The expression} = \frac{\dfrac{5}{2} \div \dfrac{15}{7} + \dfrac{5}{6}}{3\frac{7}{12} - \dfrac{10}{3} \text{ of } \dfrac{1}{60}}$$

$$= \frac{\dfrac{5}{2} \times \dfrac{7}{15} + \dfrac{5}{6}}{3\frac{7}{12} - \left(\dfrac{10}{3} \times \dfrac{1}{60}\right)}$$

$$= \frac{\dfrac{7}{2 \times 3} + \dfrac{5}{6}}{3\frac{7}{12} - \dfrac{1}{3 \times 6}}$$

$$= \frac{\dfrac{7}{6} + \dfrac{5}{6}}{3 + \dfrac{21}{36} - \dfrac{2}{36}}$$

$$= \frac{2}{3 + \dfrac{19}{36}} = \frac{2 \times 36}{3 \times 36 + \dfrac{19}{36} \times 36}$$

multiplying top and bottom by 36

$$= \frac{72}{108 + 19} = \frac{72}{127}.$$

## EXERCISES XXXIV.

1. Find the value of

(i) $\frac{1}{2} + \frac{1}{3} - \frac{1}{4}$. (ii) $10\frac{2}{5} + \frac{1}{3} + 4\frac{1}{2}$.
(iii) $10\frac{2}{5} + \frac{1}{3} - 4\frac{1}{2}$. (iv) $\frac{3}{5} + \frac{4}{7} - \frac{2}{3}$.
(v) $1 - \frac{3}{4} + \frac{7}{8}$. (vi) $2\frac{1}{2} - 3\frac{1}{4} + 1\frac{5}{8}$.
(vii) $4\frac{1}{4} - 2\frac{1}{7} - 3\frac{1}{8}$. (viii) $2\frac{1}{8} + 3\frac{1}{2} - 6\frac{5}{7}$.

2. Express in 64ths of an inch a length made up of $\frac{3}{8}''$, $\frac{5}{16}''$, $\frac{7}{32}''$, $\frac{3}{64}''$. (U.L.C.I.)

3. Find the sum of

$$\frac{a}{x^2},\ \frac{4}{x},\ \frac{3}{x^4},\ \frac{b}{x^3}. \qquad \text{(U.L.C.I.)}$$

**4.** Simplify

(i) $\frac{1}{2}+\frac{1}{3x}$. (ii) $\frac{1}{a}+\frac{1}{b}-\frac{1}{c}$.

(iii) $y-\frac{y}{x}$. (iv) $\frac{x-y}{2}-\frac{3x-2y}{3}$.

(v) $\frac{3}{xy}+\frac{1}{xz}-\frac{2}{yz}$. (vi) $\frac{a+b}{a^2}-\frac{1}{a}$.

(vii) $\frac{a-b+c}{3a^2}+\frac{a+b-c}{2a}$. (viii) $\frac{a+x}{a-x}-\frac{a-x}{a+x}$.

**5.** Simplify

(i) $(2\frac{1}{2}-3\frac{1}{4})\times\frac{3}{4}$. (ii) $\frac{\frac{1}{2}\text{ of }(3\frac{1}{4}-1\frac{5}{8})}{\frac{1}{3}\text{ of }(3\frac{1}{4}+1\frac{5}{8})}$.

(iii) $\frac{2\frac{1}{3}+\frac{1}{2}\text{ of }(\frac{3}{7}-1)}{1\frac{1}{2}\div\frac{2}{7}}$. (iv) $\frac{30a}{20b}\times\frac{15bc}{10ad}$.

(v) $\left(\frac{1}{2}+\frac{1}{3x}\right)\div\left(3+\frac{2}{x}\right)$. (vi) $\left\{a-\frac{ab}{a+b}\right\}\div\left\{b-\frac{ab}{a+b}\right\}$.

**6.** Simplify $$\frac{2\frac{1}{5}-1\frac{2}{3}}{3\frac{1}{3}+1\frac{2}{5}}\div\frac{\frac{2}{7}-\frac{1}{8}}{\frac{2}{7}+\frac{1}{8}}.$$ (Matric.)

**7.** Simplify $$\frac{\frac{1}{x}+\frac{2}{y}}{\frac{1}{x}-\frac{2}{y}}-\frac{\frac{1}{x}-\frac{2}{y}}{\frac{1}{x}+\frac{2}{y}}.$$ (Matric.)

**8.** Simplify the following expressions, and verify the results by substituting $a=1$, $b=0$, $c=-1$, $x=5$, $d=2$:

(i) $\left\{\frac{x-a}{a}-b+\frac{x+c}{c}\right\}\times\frac{c}{a-b}$.

(ii) $\left(\frac{a}{d}+\frac{d}{c}\right)\left(\frac{c}{x}-\frac{a+c}{x+d}\right)$.

**9.** Simplify the expression $\frac{y+c}{1-\frac{y+c}{\text{R}}}$, and find its value when $\text{R}=10$, $c=1$ for the following values of $y$: $y=-5$, $-3$, $-2$, $0$, $1{\cdot}5$, $2{\cdot}7$ and $3\frac{1}{3}$.

**10.** Given $u=25{\cdot}24$, $v=13{\cdot}27$, calculate $x$ to four significant figures from the equation $$\frac{1}{x}=\frac{1}{u}+\frac{1}{v}.$$ (Matric.)

# CHAPTER VIII.

## ALGEBRAIC MANIPULATION. EXPANSIONS. FACTORISATION.

**Expansion and Factorisation.**—Since $a(b+c)$ means $a$ times the sum of $b$ and $c$, *i.e.* $a$ times $b$, $+\ a$ times $c$,

$$a(b+c)=ab+ac.$$

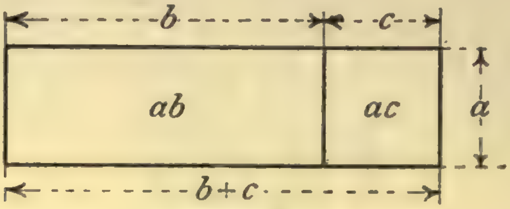


FIG. 55.

This is also evident from Fig. 55.

Consider now the expression $(x+p)(x+q)$.

This means $(x+p)$ times $x$, $+\ (x+p)$ times $q$,

viz. $$(x+p)x+(x+p)q$$

or $$x^2+px+xq+pq.$$

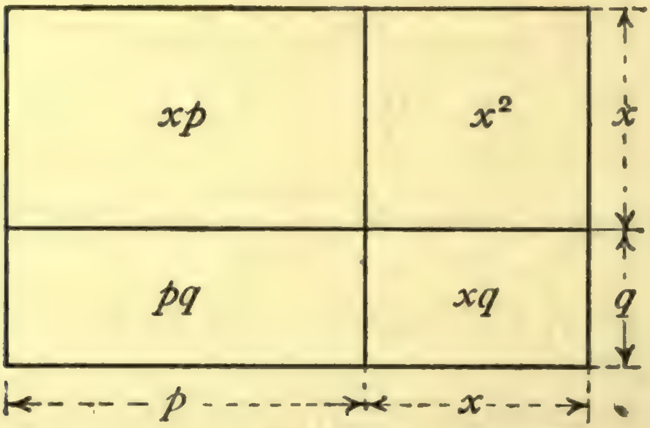


FIG. 56.

This is usually written $x^2+(p+q)x+pq$, in which the highest power of $x$ comes first, next the second highest, and so on. The last term is of course $pq\,.\,x^0$, and the expression is said to be arranged in **descending powers** of $x$, and is called the **expansion** of $(x+p)(x+q)$.

The identity $(x+p)(x+q)=x^2+(p+q)x+pq$ is illustrated by Fig. 56.

Ex. 1. *Expand* $(2a+3b)(4a-3c)$.

The expression means $(2a+3b)4a-(2a+3b)3c$,

*i.e.* $8a^2+12ab-6ac-9bc$,

*i.e.* $8a^2+6a(2b-c)-9bc$,

arranged in descending powers of a.

It frequently happens that in manipulating a formula it is necessary to find the value of an expression like $x^2-z^2$.

Ex. 2. *Suppose it is required to find the value of* $x^2-z^2$ *when* $x=72{\cdot}3$ *and* $z=51{\cdot}7$.

We might proceed as follows:

$$x^2-z^2=72{\cdot}3^2-51{\cdot}7^2\overset{\frown}{=}70^2-50^2\overset{\frown}{=}4900-2500\overset{\frown}{=}2400.$$

```
 723            517
   723            517
 -----          -----
5061           2585
 1446             517
   2169            3619
 ------         ------
522729         267289
```

$$\therefore\ x^2-z^2=5227{\cdot}29-2672{\cdot}89$$
$$=2554{\cdot}4.$$

Now suppose it is known that

$$x^2-z^2=(x+z)(x-z).$$

Then
$$x^2-z^2=(72{\cdot}3+51{\cdot}7)(72{\cdot}3-51{\cdot}7)$$
$$=124\times 20{\cdot}6$$
$$\overset{\frown}{=}2400.$$

```
124
  206
 ----
248
  744
-----
25544
```

$$\therefore\ x^2-z^2=2554{\cdot}4.$$

The amount of labour required in each of the two methods should be compared.

The process by which $x^2-z^2$ is written as $(x+z)(x-z)$ is known as **factorisation.**

When $ab+ac$ is written in the form $a(b+c)$, $ab+ac$ is said to be **factorised**, *i.e.* the expression is written as the product of its factors $a$ and $(b+c)$.

It will be seen that **expansion and factorisation are inverse operations.** Factorisation is useful in adapting formulae to logarithmic calculation.

Ex. 3. *Factorise* $a^2+2ab+3ac$.

Inspection shews that $a$ is common to each of the three terms, and we have

$$a^2+2ab+3ac=a(a+2b+3c).$$

Ex. 4. *Factorise* $ab+ay+yz+bz$.

By inspection, $a$ is common to the first two terms, and $z$ to the last two.

Hence $ab+ay+yz+bz=a(b+y)+z(y+b)$.

We have *not* yet factorised the original expression, but it will be seen that its present form contains two terms only, each of which has $(b+y)$ as a factor; and

$$a(b+y)+z(y+b)=(a+z)(y+b),$$

just as

$$ap+zp=(a+z)p.$$

Hence, finally, $ab+ay+yz+bz=(a+z)(y+b)$.

Ex. 5. *Factorise* $8a^2-3bc+2ac-12ab$.

There is no common factor to all the terms of the expression; also, the first two terms have no common factor. Now let us take the first and third terms together:

$$8a^2+2ac=2a(4a+c).$$

The remaining terms are

$$-3bc-12ab, \text{ in which } -3b \text{ is common};$$

$$\therefore\ -3bc-12ab=-3b(c+4a).$$

$\therefore$ the original expression $=2a(4a+c)-3b(c+4a)$, which are two terms having a common factor $4a+c$.

$$\therefore \text{ the expression}=(4a+c)(2a-3b).$$

Ex. 6. *Factorise* $x^2-3xz-2xy+6yz$.

$$\begin{aligned}\text{The expression}&=x(x-3z)-2y(x-3z)\\&=(x-3z)(x-2y).\end{aligned}$$

**Some Important Cases of Expansion and Factorisation.** Three cases of expansion are so important that they must be memorised; they are the following:

$$(a+b)^2 = a^2 + 2ab + b^2 ; \tag{i}$$
$$(a-b)^2 = a^2 - 2ab + b^2 ; \tag{ii}$$
$$(a+b)(a-b) = a^2 - b^2. \tag{iii}$$

These statements, when inverted, become cases of factorisation, and should also be memorised in this form. Thus,

$$a^2 + 2ab + b^2 = (a+b)^2 ; \tag{iv}$$
$$a^2 - 2ab + b^2 = (a-b)^2 ; \tag{v}$$
$$a^2 - b^2 = (a+b)(a-b). \tag{vi}$$

Identities (i), (ii) and (iii) may be shewn to be true by expansion. We have

$$\begin{aligned}(a+b)^2 &= (a+b)(a+b)\\ &= a(a+b) + b(a+b)\\ &= a^2 + ab + ab + b^2\\ &= a^2 + 2ab + b^2.\end{aligned}$$

Again,

$$\begin{aligned}(a-b)^2 &= (a-b)(a-b)\\ &= a(a-b) - b(a-b)\\ &= a^2 - ab - ba + b^2\\ &= a^2 - 2ab + b^2.\end{aligned}$$

And

$$\begin{aligned}(a+b)(a-b) &= a(a-b) + b(a-b)\\ &= a^2 - ab + ba - b^2\\ &= a^2 - b^2.\end{aligned}$$

**Geometrical Illustration of $(a+b)^2 = a^2 + 2ab + b^2$.**—Let AD (Fig. 57) be $(a+b)$ inches long, and let ABCD be a square. The area of ABCD is $(a+b)^2$ sq. inches. Let DE $= b''$. Then, since ABCD is a square, CD $= (a+b)''$, and therefore CE $= a''$. Complete the figure as shewn; the figure ABCD being thus divided up into four parts, the areas of which are marked.

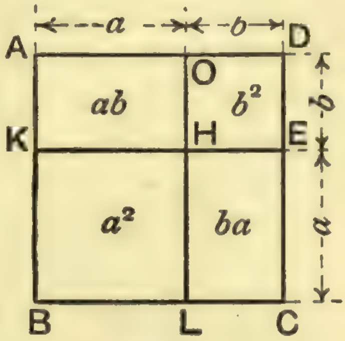


FIG. 57.

So that area of whole figure ABCD

is $(a^2 + ab + ab + b^2)$ sq. inches,

*i.e.* is $(a^2 + 2ab + b^2)$ sq. inches.

But area of ABCD is $(a+b)^2$ square inches.

$$\therefore \ (a+b)^2 = a^2 + 2ab + b^2.$$

**Geometrical Illustration of $(a-b)^2 = a^2 - 2ab + b^2$.**—Let AD (Fig. 58) be $a$ in. long, and ABCD the square on AD. Mark off DO and DE each $b$ in. long, and complete the figure as shewn.

Now KBLH = ABCD – shaded area.

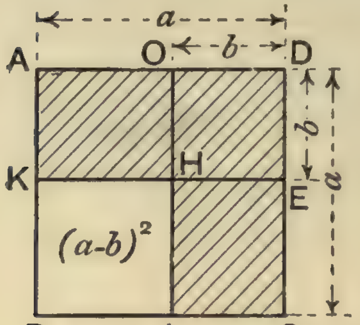


FIG. 58.

Again, the shaded area + OHED

= AKED + OLCD.

*i.e.* the shaded area

= AKED + OLCD – OHED

$= (ab + ab - b^2)$ sq. inches.

∴ KBLH = ABCD – shaded area

$= [a^2 - (ab + ab - b^2)]$ sq. inches

$= (a^2 - 2ab + b^2)$ sq. inches.

But KBLH is a square of side $(a-b)$ inches, and therefore its area is $(a-b)^2$ sq. inches.

Hence $$(a-b)^2 = a^2 - 2ab + b^2.$$

**Geometrical Illustration of $(a+b)(a-b) = a^2 - b^2$.**—Let AD (Fig. 59) be $a$ in. long, and ABCD the square on AD. From DA cut off DE $= b''$. Complete the figure as shewn.

Now ABCD – OHED $= a^2 - b^2$ sq. inches, and is the shaded part of the figure. The doubly shaded rectangle HOCL is transferred to the position marked X. Therefore the area of the shaded portion is equal to the area of the rectangle ANME, *i.e.* $(a+b)(a-b)$ sq. inches.

$$\therefore a^2 - b^2 = (a+b)(a-b).$$

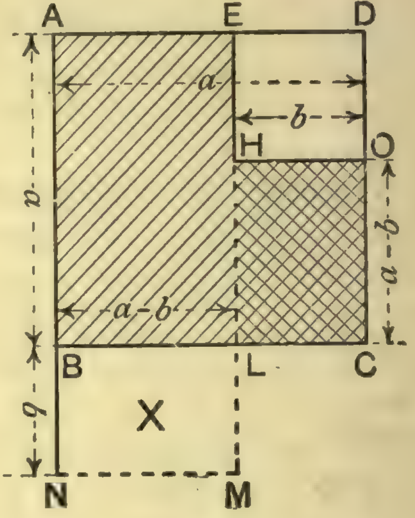


FIG. 59.

Ex. 1. *Expand* $(2x+y)^2$.

We have $$(a+b)^2 = a^2 + 2ab + b^2.$$

Writing $2x$ for $a$ and $y$ for $b$, we have

$$(2x+y)^2 = (2x)^2 + 2(2x)y + y^2$$
$$= 4x^2 + 4xy + y^2.$$

Ex. 2. *Expand* $(3a-2c)^2$.

We know $$(x-y)^2=x^2-2xy+y^2.$$

Writing $3a$ for $x$ and $2c$ for $y$, we have

$$\begin{aligned}(3a-2c)^2&=(3a)^2-2(3a)(2c)+(2c)^2\\&=9a^2-12ac+4c^2.\end{aligned}$$

Ex. 3. *Evaluate* $(0{\cdot}98)^2$.

$$\begin{aligned}(0{\cdot}98)^2&=(1-0{\cdot}02)^2\\&=1^2-2\,.\,(0{\cdot}02)\,.\,1+(0{\cdot}02)^2\\&=1-0{\cdot}04+0{\cdot}0004\\&=0{\cdot}9604.\end{aligned}$$

Ex. 4. *Factorise* $a^2+4ab+4b^2$.

The expression may be written

$$a^2+2\,.\,a\,.\,(2b)+(2b)^2,$$

and is therefore $$(a+2b)^2.$$

Ex. 5. *What are the factors of* $x^2-10$? (B.E.)

The expression is $$x^2-(\sqrt{10})^2,$$

which $$\overset{\frown}{=}x^2-(3.162)^2;$$

$\therefore$ the factors are $(x-3.162)(x+3.162)$.

## EXERCISES XXXV.

**1.** Expand

(i) $(x+y)(c-d)$. (ii) $(y-z)(p+q)$.

(iii) $(x+a)(x-k)$. (iv) $(x-a)(x+a)$.

(v) $(x+a)(x+b)(x+c)$. (vi) $(x-l)(x-m)(x-n)$;

arranging the last four in descending powers of $x$.

**2.** Expand

(i) $(2a-3b)^2$. (ii) $(3p+5q)^2$.

(iii) $(2a-3b)(2a+3b)$. (iv) $(5p-4k)(5p+4k)$.

(v) $(1000-7)(1000+7)$. (vi) $(a+b+c)^2$.

**3.** Factorise

(i) $a^2+3ab$. (ii) $6ac+a^2$.

(iii) $3a^2-9ab$. (iv) $p^2-4pq+pr$.

(v) $ax-ay+bx-by$. (vi) $x^2+px+qx+pq$.

(vii) $6x^2-5yz+2xy-15xz$. (viii) $ac+bd+ad+bc$.

**4.** Find the value of

(i) $998 \times 1002$. (ii) $(1{\cdot}01)^2$.

(iii) $(2{\cdot}97)^2$ to four significant figures.

(iv) $(0{\cdot}9997)^2$ to four significant figures.

(v) $99{\cdot}7 \times 100{\cdot}3$ to four significant figures.

(vi) $(1{\cdot}005)^2$ to four significant figures.

**5.** Factorise

(i) $9h^2 - 16k^2$. (ii) $9a^2 - 6ac + c^2$.

(iii) $97^2 - 96^2$. (iv) $(1{\cdot}03)^2 - (0{\cdot}97)^2$.

(v) $25a^2 + 10ab + b^2$. (vi) $x^2 - 5$.

(vii) $4x^2 - 7$. (viii) $1{\cdot}01x^2 - 1{\cdot}6$.

**6.** Resolve into factors

(i) $(2x-a)^2 - (x-3a)^2$. (ii) $a^2 - 4b^2$.

(iii) $a^2 - (b-c)^2$. (iv) $a^2 - 2ab - 3ac + 6bc$.

(v) $a^2 + ac - b^2 - bc$. (M.C.U.)

**Factors of Expressions of the Form $x^2 + px + q$.**—The following identities can easily be verified:

$$(x+a)(x+b) = x^2 + x(a+b) + ab,$$

$$(x-a)(x-b) = x^2 - x(a+b) + ab,$$

$$(x-a)(x+b) = x^2 - x(a-b) - ab,$$

$$(x+a)(x-b) = x^2 + x(a-b) - ab.$$

Figures illustrating each of these identities should be drawn.

The last three can be derived from the first by making obvious substitutions. Thus, in

$$(x+a)(x+b) = x^2 + x(a+b) + ab,$$

write $-a$ for $a$, and we get

$$\begin{aligned}(x-a)(x+b) &= x^2 + x\{(-a)+b\} + (-a)b \\ &= x^2 - x(a-b) - ab,\end{aligned}$$

which is the third statement above.

In an expression like $rx^2 + px + q$ there are three terms, viz. $rx^2$, $px$ and $q$. The expression is arranged in descending powers of $x$; $r$ is the **coefficient** of the highest power of $x$, viz. $x^2$; $p$ is the coefficient of $x$; $q$ is the last or **constant term.**

The identities at the beginning of this paragraph can be expressed in words as follows:

If $x$ plus **any number** be multiplied by $x$ plus **any other number**, the product equals an expression involving $x^2$, $x$ and a constant term, in which

the coefficient of $x^2$ is unity,

the coefficient of $x$ is the **sum** of the two numbers

and the constant term is the **product** of the numbers.

It may be asked how this statement includes identities other than the first. Consider the product $(x-a)(x+b)$. The two numbers here are $-a$ and $+b$. Their sum is $(-a+b)$ or $-(a-b)$; their product is $-ab$. Hence, by the statement,

$$(x-a)(x+b)=x^2-(a-b)x-ab,$$

which is the third identity. The others should be tested similarly.

Ex. 1. *Write down the value of* $(\mathrm{k}+1{\cdot}7)(\mathrm{k}-0{\cdot}8)$.

$$\begin{aligned}\text{Expression}&=\mathrm{k}^2+\mathrm{k}(1{\cdot}7-0{\cdot}8)-1{\cdot}7\times0{\cdot}8\\&=\mathrm{k}^2+0{\cdot}9\mathrm{k}-1{\cdot}36.\end{aligned}$$

Ex. 2. *Expand* $[1{\cdot}5\mathrm{x}-3{\cdot}2][1{\cdot}5\mathrm{x}+1{\cdot}7]$.

$$\begin{aligned}\text{Expression}&=(1{\cdot}5\mathrm{x})^2+(1{\cdot}5\mathrm{x})[-3{\cdot}2+1{\cdot}7]-3{\cdot}2\times1{\cdot}7\\&=2{\cdot}25\mathrm{x}^2+\mathrm{x}\,.\,1{\cdot}5(-1{\cdot}5)-3{\cdot}2-2{\cdot}24\\&=2{\cdot}25\mathrm{x}^2-2{\cdot}25\mathrm{x}-5{\cdot}44.\end{aligned}$$

The reverse process of factorising expressions like

$$x^2+(a+b)x+ab$$

can be carried out immediately.

Now, in the identity $x^2+(a+b)x+ab=(x+a)(x+b)$, note that $a$ and $b$ are two numbers, such that their sum is the coefficient of $x$ and their product the last term.

Hence, to factorise $x^2+7x+10$, we must guess two numbers whose sum is 7 and product 10; these are clearly 5 and 2.

$$\therefore\ x^2+7x+10=(x+5)(x+2).$$

Again, to factorise $x^2+(a-b)x-ab$, we must guess two numbers whose sum is $a-b$ and whose product is $-ab$, viz. $+a$ and $-b$.

$$\therefore\ x^2+(a-b)x-ab=(x+a)(x-b).$$

Ex. 1. *Factorise* $x^2-7x+12$.

$$x^2-7x+12=(x-4)(x-3),$$

since sum of $-4$ and $-3=-7$

and product of $-4$ and $-3=12$.

Ex. 2. *Find the factors of* $x^2+\left(\frac{p^2}{q}-q\right)x-pq$.

$$\text{Expression}=\left(x+\frac{p^2}{q}\right)(x-q),$$

since sum of $\frac{p^2}{q}$ and $-q$ is $\frac{p^2}{q}-q$

and product of $\frac{p^2}{q}$ and $-q$ is $-pq$.

Ex. 3. *Factorise* $x^2-0{\cdot}9x-13{\cdot}12$.

By trial, the factors of $-13{\cdot}12$, whose algebraic sum is $-0{\cdot}9$, are

$$-4{\cdot}1 \text{ and } 3{\cdot}2.$$

$$\therefore\ \text{expression}=(x-4{\cdot}1)(x+3{\cdot}2).$$

## EXERCISES XXXVI.

**1.** Write down the expansions of

(i) $(x+2)(x+3{\cdot}2)$. (ii) $(x-4{\cdot}7)(x+2{\cdot}9)$.
(iii) $(x-a)(x-1{\cdot}6)$. (iv) $(x-11{\cdot}7)(x-4{\cdot}3k)$.
(v) $(x+k)(x-1{\cdot}2l)$. (vi) $(ax+1{\cdot}5k)(ax-3{\cdot}1k)$.

**2.** Factorise

(i) $a^2+5a+6$. (ii) $x^2-7x+12$.
(iii) $p^2+3p-40$. (iv) $a^2+2a-8$.
(v) $x^2+2x-120$. (vi) $k^2-8k-65$.

**3.** Factorise

(i) $x^2+x-20$. (ii) $2a^2+8ab-24b^2$.
(iii) $x^4-13x^2+36$. (iv) $a^3-\frac{a^2}{12}-\frac{a}{12}$. (M.C.U.)

**4.** Factorise

(i) $x^2+1{\cdot}3x+0{\cdot}4$. (ii) $x^2+0{\cdot}3x-0{\cdot}4$.

(iii) $x^2-0{\cdot}3x-0{\cdot}4$.

**5.** Factorise

(i) $a^2+0{\cdot}75a-0{\cdot}25$. (ii) $a^2+1{\cdot}25a+0{\cdot}25$.

(iii) $a^2-0{\cdot}75a-0{\cdot}25$.

**6.** Factorise

(i) $x^2-1{\cdot}9x-5{\cdot}1$. (ii) $x^2+1{\cdot}9x-5{\cdot}1$.

(iii) $x^2-4{\cdot}9x+5{\cdot}1$.

**7.** Factorise

(i) $65-18x+x^2$. (ii) $x^2y^2-20xy+99$.

(iii) $132-p^2q^2-pq$. (iv) $x^2+12xy-45y^2$.

(v) $x^2-130y^2+3xy$.

# CHAPTER IX.

## AVERAGES. PERCENTAGES. RATIO AND PROPORTION.

**Averages.**—Every cricketer knows the meaning of the word **average** as applied to his score. During the cricket season, newspapers frequently print tables headed 'batting averages,' thus:

| Name of Batsman. | Innings. | Runs. | Most in Innings. | Times Not Out. | Average. |
|---|---|---|---|---|---|
| *a* | 13 | 736 | 175 | 1 | 61·33 |
| *b* | 13 | 717 | 152* | 1 | 59·73 |
| *c* | 17 | 950 | 127 | 1 | 59·37 |
| *d* | 17 | 924 | 143* | 1 | 57·75 |
| *e* | 15 | 794 | 211 | 1 | 56·71 |

* Means 'not out.'

The numbers in the last column are obtained by dividing the number of runs by the number of completed innings, and represent the number of runs which the batsmen must have obtained in each innings in order to make the same total.

In the same way, **the average of any series of numbers is obtained by dividing the sum of the numbers by the number of numbers.**

Symbolically, the average of $a$, $b$, $c$, $d$, etc., where there are $n$ numbers, is:

$$\frac{a+b+c+d+\text{etc.}}{n}.$$

This can evidently be expressed in the equivalent form:

**n × (the average number) = the sum of the numbers.**

Ex. 1. *From the following table find the average number of lives lost at sea during the given period:*

| Year - | 1900 | 1901 | 1902 | 1903 | 1904 |
|---|---|---|---|---|---|
| Lives lost | 1537 | 1277 | 1179 | 1239 | 1068 |

Total number of lives lost = 6300.

Number of years = 5.

∴ average number of lives lost per year

$$= \frac{6300}{5} = 1260.$$

(Notice the use of the word 'per,' to indicate division.)

Ex. 2. *The average of a certain set of* $a$ *numbers is* $m$; *of another set of* $b$ *numbers is* $n$; *of a set of* $c$ *numbers is* $p$. *There are also two other numbers* $x$ *and* $y$. *What is the average of all these numbers?*

The sum of the $a$ numbers whose average is $m$ is $am$.
" " $b$ " " $n$ " $bn$.
" " $c$ " " $p$ " $cp$.

∴ the sum of all the numbers including $x$ and $y$ is

$$am + bn + cp + x + y.$$

The number of numbers is $a + b + c + 2$.

∴ the average of all the numbers is $\dfrac{am + bn + cp + x + y}{a + b + c + 2}$.

## EXERCISES XXXVII.

**1.** Find the average of the following:

6·281, 0·0024, ·381, 100, 0, 6·282.

Divide your answer by 4·2827, correct to two places of decimals. (M.C.U.)

**2.** Find the average of the following numbers:

3·142, 3·140, 3·148, 3·138, 3·145, 3·139. (M.C.U.)

**3.** The following are the premiums charged by several insurance companies for a whole life assurance of £100, the assured being aged 21 on taking out the policy:

| | |
|---|---|
| British Equitable | £1. 18*s.* 10*d.*, |
| Clergy Mutual | £1. 16*s.* 0*d.*, |
| General Accident | £2. 0*s.* 2*d.*, |
| National Provident | £2. 0*s.* 3*d.*, |
| North British and Mercantile | £1. 19*s.* 1*d.*, |
| Alliance | £2. 0*s.* 9*d.* |

Find the average premium charged, to the nearest penny.

**4.** The following table shews the amounts deposited in the Post Office Savings Bank over a period of 10 years. Find the average amount of the annual deposit, to the nearest £1000.

| Year. | Deposited during Year. | Year. | Deposited during Year. |
|---|---|---|---|
| | £ | | £ |
| 1901 | 41,452,051 | 1906 | 43,980,579 |
| 1902 | 42,217,003 | 1907 | 44,217,288 |
| 1903 | 40,857,206 | 1908 | 44,770,782 |
| 1904 | 40,612,967 | 1909 | 45,300,525 |
| 1905 | 42,300,617 | 1910 | 46,205,870 |

**5.** $n$ fish weigh $m$ lb. between them. Of these, $b$ average $c$ lb. each. What is the average weight of the remainder? (Matric.)

**6.** The average of a certain set of $p$ numbers is $a$, and that of another set of $q$ numbers is $b$. Find an expression for the average of all the numbers taken together.

The populations of two towns are 107,509 and 189,160; their birth-rates per thousand are 27·9 and 25·7. Find to the same degree of exactness the birth-rate for the two towns taken together. (Matric.)

**7.** In an eight-oared boat, the weight of the whole crew was 92 stone 7 lb. What is the average weight of each oarsman? (14 lb. = 1 stone.)

**8.** A does a piece of work in 3½ days, B in 4 days and C in 5 days 5 hours, the working day being 8 hours. The day's work of D is the average of A, B, C. How many days does D require to do the work?

**Percentages.**—$x$ **per cent.** means $x$ **divided by** 100, and is written $x$ %. Thus 10 % means 'ten per cent.', *i.e.* 10/100, or 'one in ten.'

The efficiency of a power-plant, *i.e.* the number obtained by dividing the useful output by the energy supplied, is generally expressed as a percentage. Thus, if it is said that the net efficiency of a plant is 12 %, the meaning is that for every 100 units of energy supplied, 12 units are actually available.

| | |
|---|---|
| Suppose the calorific value of the fuel supplied to a Diesel engine be | 100 |
| Then, on the average, the loss in the exhaust, cooling, etc. is | 55 |
| ∴ the indicated horse-power (I.H.P.) is | 45 |
| Average loss in friction, air-pump, etc. is | 11 |
| ∴ the brake horse-power (B.H.P.) is | 34 |
| If the engine is driving a generator whose efficiency is 91 %, the loss in the generator would be $\frac{9}{100}(34)$, or roughly | 3 |
| ∴ the combined efficiency of engine and generator at full load is | 31 |

Or, in other words, the combined efficiency is 31 %.

For comparison it may be stated that the combined efficiency of the best type of steam-engine, condenser and alternator is only 12 %.

The following typical examples illustrate the method of working percentages.

Ex. I. *Gun-metal consists of* 8 *parts copper*, 1 *part tin. Express each as a percentage of the whole.*

| | | | | | | |
|---|---|---|---|---|---|---|
| In | 9 parts of metal there are | | 8 | parts copper, | 1 | part tin. |
| ∴ in | 1 | „ | „ | $\frac{8}{9}$ | „ | $\frac{1}{9}$ „ |
| ∴ in | 100 | „ | „ | $\frac{800}{9}$ | „ | $\frac{100}{9}$ „ |

*i.e.* 88·89 parts copper and 11·11 parts tin.

That is to say, there are 88·89 parts per 100 of copper and 11·11 parts per 100 of tin.

Otherwise, gun-metal consists of 88·89 % copper and 11·11 % tin.

Ex. 2. 25 *per cent. of the population of Ireland live in Leinster;* 10 *per cent. of the population of the British Isles, viz.* 43 *millions, live in Ireland. How many people live in Leinster?*

The total number of people living in Ireland

$$=10\,\% \text{ of } 4{\cdot}3\times10^7=\tfrac{1}{10}\times4{\cdot}3\times10^7$$
$$=4{\cdot}3\times10^6.$$

The total number of people living in Leinster

$$=25\,\% \text{ of the number living in Ireland}$$
$$=\tfrac{25}{100}\times4{\cdot}3\times10^6=\tfrac{1}{4}\times4{\cdot}3\times10^6$$
$$=1{\cdot}075\times10^6$$
$$=1075000.$$

Ex. 3. *When* $x$ *is small we may take* $(1+x)^n$ *as being nearly equal to* $1+nx$. *What is the percentage error in this when* $x=0{\cdot}01$ *and* $n=2$? (B.E.)

We have

$$\begin{array}{l|l}
(1+x)^n & \quad 1+nx \\
=(1+0{\cdot}01)^2 & =1+2(0{\cdot}01) \\
\qquad\begin{array}{r} 101 \\ \underline{101} \\ 101 \\ \underline{101} \\ 10201 \end{array} & \\
=1{\cdot}0201 & =1{\cdot}02
\end{array}$$

The error is $0{\cdot}0001$ in $1{\cdot}0201$;

$$\therefore \text{ the error is } \frac{0{\cdot}0001}{1{\cdot}0201} \text{ in } 1,$$

$$\textit{i.e.}\quad \frac{0{\cdot}01}{1{\cdot}0201} \text{ in } 100.$$

Roughly the error per cent. is $0{\cdot}01$.

$$\begin{array}{r|l}
 & \overline{\phantom{0}\;9803} \\
10201 & 100000 \\
 & \phantom{0}\;8191 \\
 & \phantom{000}\;30
\end{array}$$

Error $=0{\cdot}0098\,\%$.

## EXERCISES XXXVIII.

**1.** On board a ship there were 1312 men, 514 women and 132 children. State these as percentages of the total number of persons. (B.E.)

**2.** The composition of Babbit's metal is to be 4 parts copper, 8 antimony and 96 tin. Express these as percentages, and find the weight of each material required to make 150 lb. of the metal.

**3.** An alloy consists of 38 lb. of tin and 59 lb. of lead. What is its percentage composition?

**4.** A lump of alloy contains 3·41 lb. of copper, 0·97 lb. of zinc, 0·31 lb. of lead and 0·26 lb. of other material: what are the percentages of copper, zinc and lead in the alloy? (B.E.)

**5.** The workers in a factory are 235 men, 171 women and 29 children. State these as percentages of the whole number of workers. (B.E.)

**6.** A sample of coal weighing 3·5 lb. was found on analysis to be made up of 2·631 lb. carbon, 0·244 lb. hydrogen, 0·207 lb. oxygen, 0·143 lb. nitrogen; the rest being ash. Find the percentage composition of the coal. (B.E.)

**Ratio.**—Fig. 60 shews on two distinct scales drawings of a cast-iron plate, length 6″ and breadth 3″. In the first the scale is 1 cm. = 1″ and the second 4″ = 1 foot.

In (*a*) AB = 6 cm. and AD = 3 cm., and therefore AB contains AD twice.

Again, in (*b*) AB = 2″ and AD = 1″, and therefore AB contains AD twice. Whatever scale is used for representing the plate 6″ × 3″, AB will always contain AD twice. Mathematically it is said that the **ratio** of AB to AD is constant and equal to 2.

This conception of ratio can only be used when comparing two quantities of the same kind; as, for example, two lengths, two weights, two velocities, two coins, etc. It is meaningless to speak of the ratio of one elephant to one apple; on the other hand, the ratio of the weight of the elephant to the weight of the apple, or of the volume of the elephant to the volume of the apple, has a meaning.

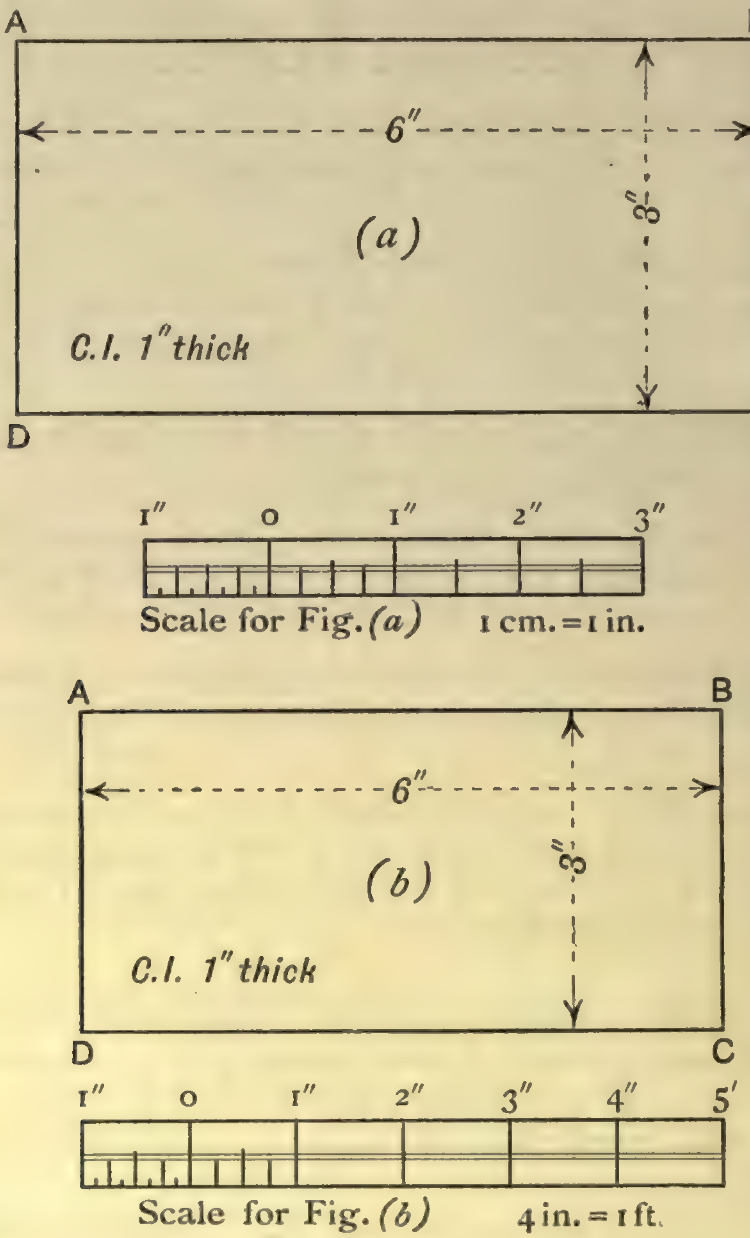


Fig. 60.

**By the ratio of one quantity to a second is meant the number of times the first quantity contains the second.**

Symbolically the ratio of $a$ to $b$ is the number of times $a$ contains $b$, *i.e.* the quotient obtained by dividing $a$ by $b$, or $\frac{a}{b}$.

Thus, the ratio of 4 to 2 is $\frac{4}{2}$, or 2.

The ratio of 3 to 6 is $\frac{3}{6}$, or $\frac{1}{2}$, or 0·5.

The ratio of 1·2 to 0·7 is $\frac{1{\cdot}2}{0{\cdot}7}$, or 1·714 to 4 significant figures.

Again, the ratio of 2*s.* to £1 is *not* $\frac{2}{1}$, since 2*s.* does not contain £1 twice, but is $\frac{2}{20}$, or 0·1.

The ratio of 1 yard to 1 mile is $\frac{1}{1760}$.

The ratio of 2 lb. to 4 oz. is $\frac{2\times 16}{4}$, or 8.

To obtain the ratio of one quantity to another, care must be taken to express each in the same units.

The ratio of *a* to *b* is frequently written $a:b$, and is read *a* is to *b*.

## EXERCISES XXXIX.

Find the following ratios:

1. 4*s.* 6*d.* to 15*s.* 2. $1\frac{1}{2}$ miles : 1980 yards.
3. 3 fr. : 10*s.* given that 25 fr. = £1.

Express as decimals to not more than four significant figures:

4. 2 lb. : 4 oz. 5. 1 ton : 600 lb. 6. 2 Kg. : 0·63 Kg.
7. 20 feet per sec. : 10 miles an hour.
8. 1 ft. per sec. per sec. : 40 ft. per sec. per hour.
9. 75 lb. per sq. in. : 4500 lb per sq. foot.
10. 8·5 gm. per c.c. : 13·6 gm. per c.c.
11. 4·5 tons : 175 kilos.
12. £240 per ton : 8*s.* per ounce.

**Proportion.**—Statements like the following are common: "The area of a 9″ board is proportional to its length"; "The area of a circle is proportional to the square of its diameter"; "The tensile strength of an iron rod is proportional to its cross-sectional area"; "The horse-power of an engine of given size, running at a given speed, is proportional to the steam pressure"; "The volume of a given quantity of gas at constant temperature is inversely proportional to the pressure"; and so on. In each the word **proportional** is used, and the meaning of this word can be arrived at by considering the first statement.

The formula for the area of a 9″ board is

$$\text{Area in sq. feet} = \text{length in feet} \times \tfrac{3}{4}.$$

Hence, for *any* 9″ board, the area in sq. feet, divided by the length in feet, is always $\frac{3}{4}$, *i.e.* the ratio of the area in sq. ft. to the length in ft. is always 0·75. This fact, that for any number of 9″ boards the ratio $\dfrac{\text{area in sq. feet}}{\text{length in feet}}$ is constant, is expressed by saying "The area of a 9″ board is **proportional** to its length."

Thus, **Proportion indicates the equality of two or more ratios.** Symbolically, the statement that the numbers $p$, $q$, $r$, $s$ are proportional to the numbers $x$, $y$, $z$, $w$ means that

$$\frac{p}{x} = \frac{q}{y} = \frac{r}{z} = \frac{s}{w}.$$

So the statement '*a*, *b*, *c*, *d* are in proportion' or '*a*, *b*, *c*, *d* are proportionals' means

$$\frac{a}{b} = \frac{c}{d}.$$

In the rectangles (*a*), (*b*) of Fig. 60, AB : AD is the same for each rectangle. Therefore the two sides of one rectangle and the corresponding sides of the other are in proportion.

The circumference of a circle is given by the formula:

$$\frac{c}{d} = \pi,$$

where $c$ is the circumference and $d$ the diameter, each measured in the same units. An equivalent mathematical statement is that "the circumference of a circle is proportional to its diameter," the constant of proportion being $\pi$.

If a body be let fall from rest, $v = 8\sqrt{s}$ is an approximate formula connecting its speed $v$ ft. per sec. with the distance $s$ feet through which it has fallen. The formula, written in the form $v/\sqrt{s} = 8$, shews that an equivalent mathematical statement is that "the speed of a falling body is proportional to the square root of the distance it has fallen from rest." If the units of length and time are 1 foot and 1 second respectively, the constant of proportion is 8.

**Variables and Constants.**—In the last illustration, the quantities $v$ and $s$ are changing continually; the further the body has fallen, the greater will be its speed. But no matter what the distance $s$ feet through which it has fallen may be, the *corresponding* speed $v$ ft. per sec. is always such that $\frac{v}{\sqrt{s}}=8$.

$v$ and $s$ are called **variables**; the number obtained by dividing $v$ by $\sqrt{s}$, in this instance 8, is called a **constant** for the purpose of distinction.

So in the case of the circle, the diameter may be pictured as continually increasing, and in consequence the circumference as growing, so that the circumference and the diameter are variables; while if any particular circumference be divided by its corresponding diameter, we always obtain the constant $\pi$.

Suppose a cyclist free-wheels down a hill at a steady rate of 18 miles an hour. It is clear that he goes 0·3 mile in the first minute, 0·6 mile in the first two minutes, 0·9 mile in the first three minutes, and so on. The formula for the distance S miles which the cyclist travels in $t$ minutes, is $S=0{\cdot}3\times t$. Clearly S and $t$ are variables, and are such that $\frac{S}{t}$ always equals 0·3, a constant. Consequently S is proportional to $t$, the constant of proportion being 0·3.

**Variation.**—In the above and similar cases, in order to bring out the fact that continually varying quantities are being dealt with, it is better to replace the words "is proportional to" by the words "varies directly as."

In the case of the cyclist, for instance, the distance travelled, S miles, varies directly as the time taken, $t$ minutes; the constant of variation being 0·3.

It should be clear now that there is a close connection between proportion and variation. The general case of either proportion or variation may be expressed symbolically as follows:

**If p be any varying quantity, and q depends on p in such a way that (1) q is proportional to p, or (2) q varies directly as p, we may write at once**

$$\frac{q}{p}=k,$$

**where k is a constant whose numerical value may be determined by substituting the corresponding values of p and q in the equation in any particular case.**

To save space, the words "varies directly as" may be replaced by the sign $\propto$.

Thus, $x \propto y$ means $x$ varies directly as $y$, and consequently that $\frac{x}{y}$ always has the same value.

Again, $x \propto y^2$ means $\frac{x}{y^2} =$ a constant, and is read $x$ varies directly as $y^2$.

$x \propto \frac{1}{y}$ means $\frac{x}{1/y}$ has the same value always;

*i.e.* $xy$ is a constant, and is read as $x$ varies directly as $\frac{1}{y}$, or more frequently as **x varies inversely as y.**

It may happen that the variable $x$ depends on several other variables; *e.g.* $x \propto yz^2w$. This means $\frac{x}{yz^2w}$ always has the same value.

An important case is given by the formula $H = \frac{PLAN}{33000}$, connecting the indicated horse-power of an engine with P, the steam pressure in lb. per sq. inch, A the area of the piston in sq. in., L the length of stroke in feet and N the number of working strokes per minute. In designing an engine, P, L, A and N are at disposal, and may therefore vary, so that H varies in consequence; and hence $H \propto PLAN$, the constant of variation being $\frac{1}{33000}$.

Ex. 1. *The horse-power of the engines of a ship being proportional to the cube of the speed; if the horse-power is* 2000 *at a speed of* 10 *knots, what is the power when the speed is* 15 *knots?* (B.E.)

Let H denote horse-power exerted when the speed is V knots.

Then $$H \propto V^3;$$

*i.e.* $$\frac{H}{V^3} = k \text{ (a constant).}$$

Now $H=2000$ when $V=10$;

$$\therefore \ \frac{2000}{10^3}=k; \qquad \textit{i.e. } 2=k.$$

Hence $\frac{H}{V^3}=2$ always, for the particular ship considered.

Then $\frac{H}{15^3}=2$, where $H$ is the horse-power when the speed is 15 knots; *i.e.* $H=2\times15^3=6750$.

There will be no confusion due to the fact that the symbol $H$ is used at one time for an unknown, and is replaced at another by the number 2000, if it is remembered that $H$ stands for a varying quantity, and only has the value 2000 when $V$ is 10.

Ex. 2. *If* $y \propto \sqrt{x}$, *and if* $y=5$ *when* $x=3$, *state the true formula connecting* $x$ *and* $y$. *What is* $y$ *if* $x$ *is* 9? (B.E.)

We have $$\frac{y}{\sqrt{x}}=k. \ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

Now $y=5$ when $x=3$.

$\therefore \ \frac{5}{\sqrt{3}}=k$, and formula (1) becomes $\frac{y}{\sqrt{x}}=\frac{5}{\sqrt{3}}$, a numerical formula connecting $x$ and $y$.

When $x$ is 9, the corresponding value of $y$ is given by

$$\frac{y}{\sqrt{9}}=\frac{5}{\sqrt{3}};$$

*i.e.*

$$y=\frac{5}{\sqrt{3}}\times\sqrt{9}$$

$$=\frac{15}{\sqrt{3}}\simeq\frac{15}{1{\cdot}732}\simeq 8{\cdot}66.$$

$$\begin{array}{r|r} & 8660 \\ \hline 1\cancel{7}\cancel{3}\cancel{2} & 15000 \\ & 1144 \\ & 105 \\ & 1 \end{array}$$

## EXERCISES XL.

1. If $y$ is proportional to the square root of $x$, and if $y$ is 3·5 when $x$ is 4, express $y$ in terms of $x$. What is $y$ when $x$ is 25? (B.E.)

2. In the following give your reasons, a 'yes' or 'no' alone being of no value.

(*a*) 15 tons of coal cost £16. 10*s*. and 9 tons cost £9. 18*s*. Is the price proportional to the quantity?

(*b*) The base of one equilateral triangle is 2″ and its area

0·866 sq. in. The base of another is 4″ and its area 3·464 sq. in. Are the areas proportional to the bases?

(*c*) £50 put out at interest amounts in two years to £54, in three years to £56 and in five years to £60. Is the amount proportional to the time? (C.S.C.)

**3.** Two mats are of the same shape. One is 7 ft. 6 in. long and 4 ft. 3 in. wide. The other is 3 ft. 9 in. wide. How long is it?

**4.** A cubic centimetre of water weighs 1 gram.

1 kilogram = 2·205 lb.
1 gallon = 4·546 litres.

Are these statements consistent with the additional statement, 1 gallon of water weighs 10 lb.?

**5.** Assume that 1 yard = 0·9144 metre. Find the acceleration of a falling body in cm. per sec. per sec., given that it is 32·2 feet per sec. per sec.

**6.** Given that the standard atmospheric pressure is 14·7 lb. per sq. inch, express it in kilograms per sq. cm., assuming 1 lb. = 453·6 gm. and 1 foot = 0·3048 metre.

**Similar Triangles.**—Drawings will shew that equiangular triangles—that is, triangles whose angles are equal—are of the same shape. Such triangles are called **similar triangles**. They are practically drawings of one triangle on different scales.

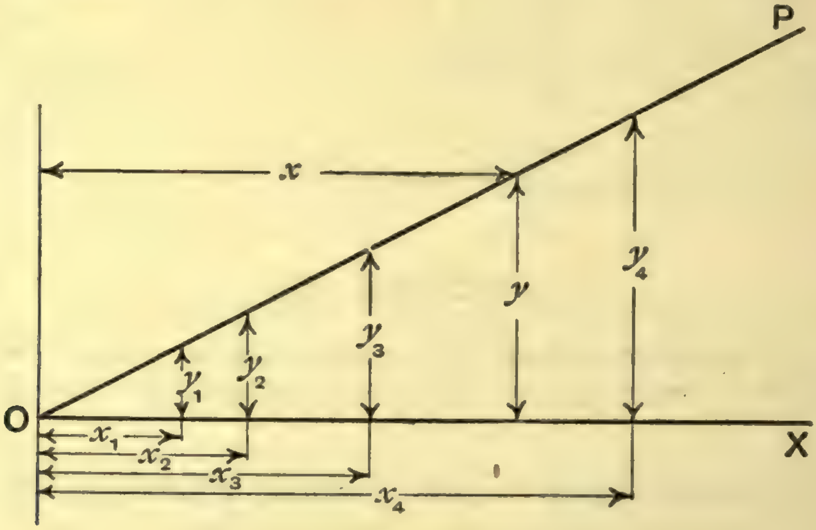


FIG. 61.

**Straight-line Graph through Origin.**—Fig. 61 shews five similar triangles having a common angular point O and a common base line OX; consequently the vertices lie on the same straight line OP.

From the figure we have

$$\frac{y_1}{x_1}=\frac{y_2}{x_2}=\frac{y_3}{x_3}=\frac{y_4}{x_4}=\text{etc.}$$

If the triangles are right angled, as shewn in Fig. 61, it is clear that the ratio of the height $y$ of any triangle to its base $x$ always has the same value; or shortly, $\frac{y}{x}$ is constant.

The application of the straight-line graph is evident.

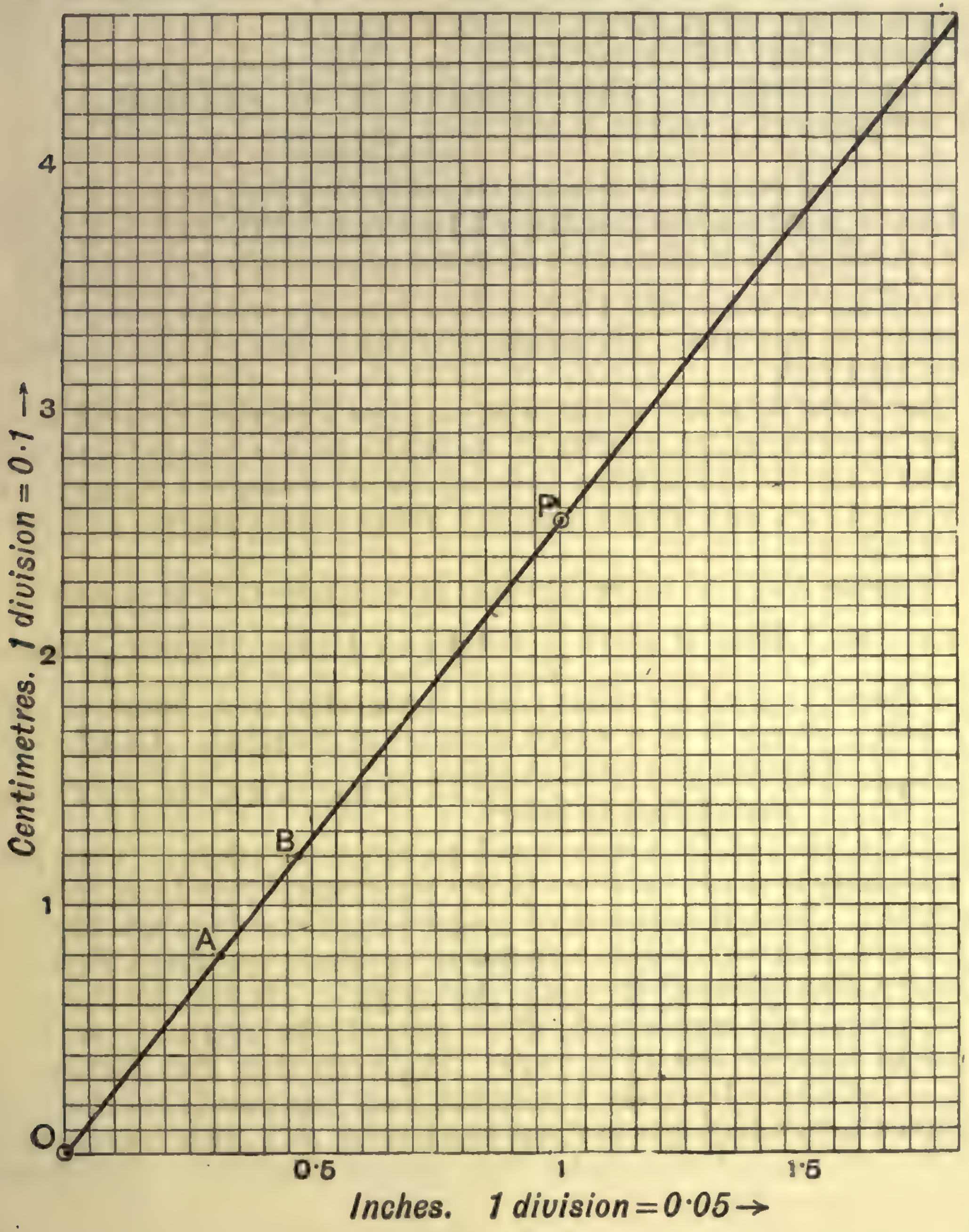


FIG. 62.

Ex. 1. *It is known that* $1'' = 2{\cdot}54$ *cm. Write down in inches the following lengths in cm.*: 0·8, 1·2, 1·5, 1·8, 2·3, 2·7, 3·1, 3·8, 4·2.

The following method is the quickest and most convenient for performing the calculation. On the vertical axis (Fig. 62) take one division to represent 0·1 cm., and on the horizontal axis two divisions to represent 0·1 in. Mark the point P, representing the fact $1'' = 2{\cdot}54$ cm. Join the origin O to P, and produce OP.

If any point on OP be taken, the horizontal distance of that point from the vertical axis gives the number of inches corresponding to the number of centimetres represented by the height of the point above the horizontal axis. Thus, the point A shews that 0·8 cm. $\simeq$ 0·32 in. The point B shews that 1·2 cm. $\simeq$ 0·47 in. The conversion of the remaining lengths in centimetres to inches is left as an exercise.

Ex. 2. *A motor car travels* 18 *miles in* 54 *minutes; a second* 19 *miles in* 56 *minutes; a third* 23 *miles in* 1·20 *hours. Arrange the speeds of these three cars in order of magnitude.*

In Fig. 63 times are plotted horizontally, one division representing 2 minutes; distances vertically, two divisions representing 1 mile.

The *point* $A_1$ refers to the first car, and indicates that 18 miles were covered in 54 minutes; $A_2$ refers to the second car, and shews that 19 miles were covered in 56 minutes; $A_3$ refers to the third car, and shews that 23 miles were covered in 1 hour 12 minutes.

Suppose any car to be moving at a steady rate of $v$ miles an hour. In $\frac{1}{2}$ hour it goes $\frac{v}{2}$ miles; in $\frac{1}{3}$ hour it goes $\frac{v}{3}$ miles; in 2 hours it goes $2v$ miles; and, generally, in $x$ hours it goes $vx$ miles, where $x$ is a whole number or fraction. In all cases the ratio $\dfrac{\text{distance travelled in miles}}{\text{time taken in hours}}$ is the same, viz. $v$; *i.e.* the distance covered is proportional to the time taken. This proportionality is really involved in the statement "steady or uniform motion." Now, if two sets of proportional numbers are plotted, we know that the points representing these numbers lie on a straight line passing through the origin. Hence, in the particular example considered, if we join O, the origin, to $A_1$ by a straight line, we obtain a graph $OA_1$, such that the distance of any point on it above the horizontal axis represents the number of miles over which the first car has passed in a time represented by the distance of that point from the vertical axis. Similarly, for the other two cars.

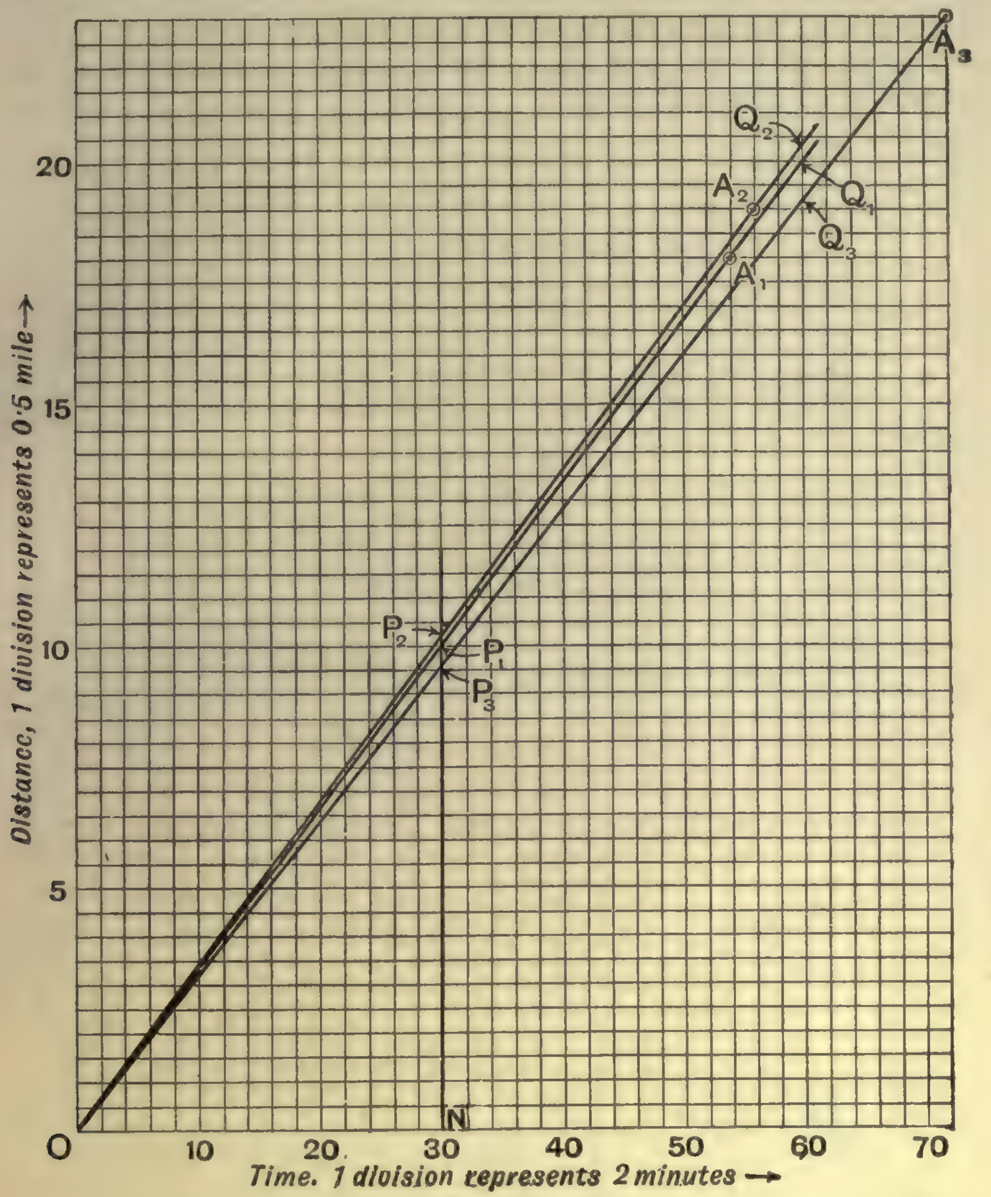


FIG. 63.

Thus, the points $P_1$, $P_2$, $P_3$, shew that in $\frac{1}{2}$ hour the first, second and third cars travelled distances represented respectively by $NP_1$, $NP_2$, $NP_3$.

Hence it is clear that of the three cars the second is travelling most rapidly and the third most slowly.

On the assumption that the motion is steady, the speeds can be arranged in order of magnitude by comparing the distances passed over in any, the same, time interval. But evidently the greater the speed the steeper will be the straight line representing the motion. Hence, in some way or other, **the slope of the graph is a measure of the speed,** and speeds can be compared by plotting the data as at the points $A_1$, $A_2$, $A_3$, without making any calculations.

The points $Q_1$, $Q_2$, $Q_3$ shew that the speeds of the cars are respectively 20 miles per hour, 20·36 miles per hour and 19·17 miles per hour approximately.

Ex. 3. *The table gives the volumes* V *c.c. of spheres of radii* r *cm.*:

| V | 7·24 | 20·58 | 65·45 |
|---|---|---|---|
| r | 1·2 | 1·7 | 2·5 |

*Shew graphically that* $V \propto r^3$, *and find the formula connecting* V *and* r.

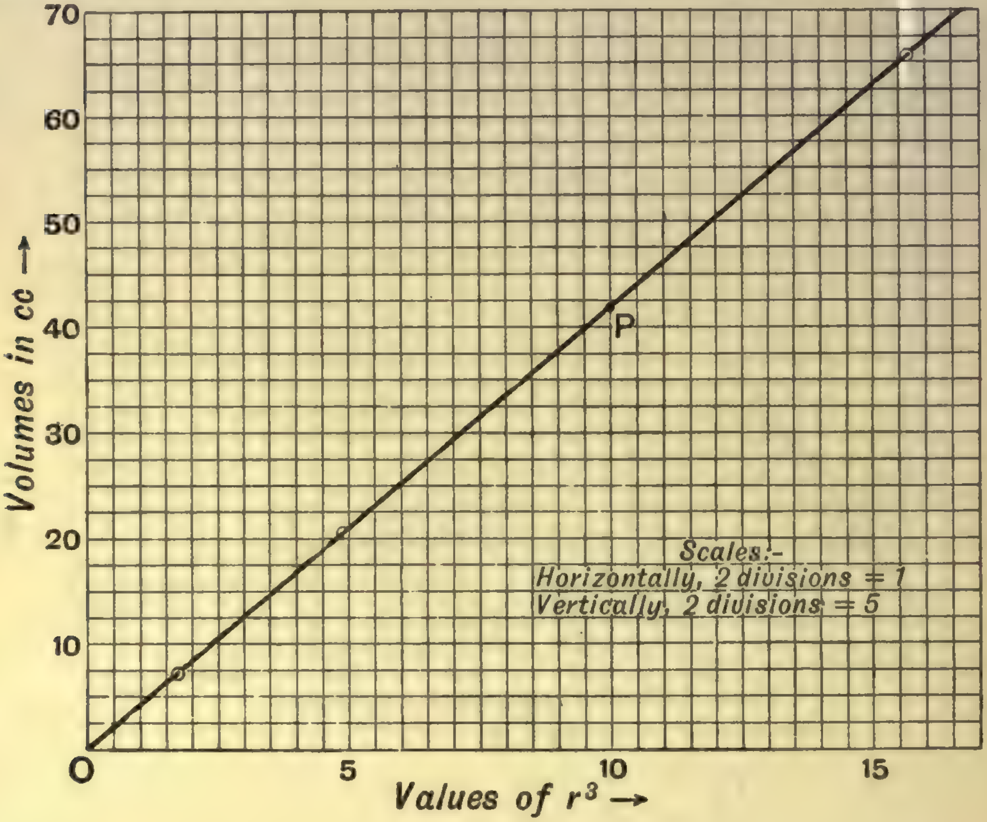


FIG. 64.

Since it is required to shew that $V \propto r^3$, the values of V must be plotted against values of $r^3$:

| V | 7·24 | 20·58 | 65·45 |
|---|---|---|---|
| $r^3$ | 1·73 | 289<br>17<br>289<br>2023<br>4913 4·91 | $62\cancel{5}$<br>25<br>1250<br>313<br>1563 15·63 |

These numbers are plotted in Fig. 64, and evidently lie on a straight line passing through the origin. Hence $V \propto r^3$.

When $r^3 = 10$, $V \simeq 41.9$, as shewn by the point P.

$$\therefore \frac{V}{r^3} = \frac{41.9}{10}.$$

$$\therefore V \simeq 4.19 r^3.$$

If any other point on the graph be taken to give particular values of V and $r^3$, the same formula will be obtained.

*Note.*—**The volume of a sphere is given by the formula**

$$V = \frac{4}{3}\pi r^3.$$

## EXERCISES XLI.

**1.** What modification in the scales of the graph shewn in Fig. 62 would make it possible to convert into inches lengths lying between 0 and 45 cm.?

**2.** Convert to cm. the following readings in inches:

0·24, 0·77, 1·8, 1·25, 10, 17·8.

**3.** A given weight of gas is heated, the pressure remaining constant. The volume V c.c. and the corresponding absolute temperature T are given in the table:

| V | 100 | 105·5 | 137 |
|---|---|---|---|
| T | 273 | 288 | 373 |

Test graphically if the volume is proportional to the absolute temperature. What will be the volume of the same weight of gas at 100, 200, 300 absolute?

**4.** On the squared paper draw a straight-line graph shewing the relation between the diameter and the circumference of any circle, from 0 to 3 in. diameter, given that the circumference of a circle 3 in. diameter = 9·42 in.

Plot the circumferences and diameters *full size.*

From the graph find the circumference of a circle 1·8 in. diameter, and the diameter of a circle whose circumference is 8·2 in. (U.L.C.I.)

**5.** The following table gives the circumferences and corresponding diameters of certain circles. Shew graphically that the circumference of a circle is proportional to its diameter, and deduce the formula connecting them.

| Diameter in inches | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| Circumference in inches - - | 3·14 | 6·28 | 9·42 | 12·57 | 15·71 | 18·85 |

**6.** Assume that £1 is equivalent to 4·867 United States gold dollars. Draw a graph from which the English equivalent of American money may be obtained by inspection.

What are the English equivalents, to the nearest penny, of (*a*) 1 dollar; (*b*) \$2.50 (there are 100 cents in a dollar); (*c*) \$12.40?

Express in dollars and cents

(*a*) £1. 6*s*. 8*d*.; (*b*) 10*s*.; (*c*) 4*s*. 2*d*.; (*d*) £5. 5*s*. 6*d*.

**7.** Determine which of the following express-runs is the fastest:

(i) 36 miles in 39 minutes.
(ii) $25\frac{1}{4}$ miles in 28 minutes.
(iii) $48\frac{1}{2}$ miles in 55 minutes.
(iv) $34\frac{1}{8}$ miles in 36 minutes. (C.S.C.)

**8.** The table shews the number of ships passing through the Suez Canal and their tonnage for certain years. Find graphically in which year the average tonnage per ship was (i) greatest, (ii) least.

| Year. | Number of Ships. | Tonnage. |
|---|---|---|
| 1901 | 3699 | 15,163,233 |
| 1902 | 3708 | 15,694,359 |
| 1903 | 3761 | 16,615,309 |
| 1904 | 4237 | 18,661,092 |

**9.** A, B, C, D are four stations on a railway. AB = 10, BC = 10, CD = 8 miles. The following is an extract from a time table:

| | Up. | | Down. |
|---|---|---|---|
| A | dep. 7.57 a.m. | D | dep. 8.29 a.m. |
| B | arr. 8.15 ” | B | — |
| | dep. 8.18 | | |
| C | arr. 8.37 ” | C | — |
| | dep. 8.40 | | |
| D | arr. 8.55 ” | A | arr. 9.10 a.m. |

Plot in one diagram graphs to shew the positions of both trains at any time between those given, assuming that they run uniformly between the stations. When and where do they pass one another? (C.S.C.)

**10.** A signals 145 words in 23 minutes, B 94 words in 15 minutes, C 133 words in 21 minutes. Who is the fastest signaller? Shew on the diagram the graph corresponding to a rate of 6 words per minute. (C.S.C.)

**11.** Oxford is $63\frac{1}{2}$ miles from London by rail. A train starting from Oxford arrives at London at 8.48 p.m. Another train, starting from London at 7.30 p.m., passes the first 31 miles from London, and arrives at Oxford at 8.45 p.m. Supposing both trains to run at uniform speeds without stopping, find the rate at which the first train travels and the time at which it starts from Oxford. (C.S.C.)

**12.** In 1908 an income-tax of 1*s.* in the pound was subject to the abatements given in the following table:

| Exceeding. | Not Exceeding. | Abatement. |
|---|---|---|
| — | £160 | No tax |
| £160 | 400 | No tax on first £160 |
| 400 | 500 | " " 150 |
| 500 | 600 | " " 120 |
| 600 | 700 | " " 70 |
| 700 | — | No abatement |

Draw a graph to shew the tax on all incomes from £150 to £900.

If this tax were replaced by one of 1*s.* 8*d.* in the pound on the excess of each income over £300 (those not exceeding £300 being free of tax), between what limits must be the income of a man who would be benefited? (C.S.C.)

# CHAPTER X.

## MENSURATION OF CIRCLE, CYLINDER AND CONE

**Definition of $\pi$.**—The ratio of the circumference of a circle to its diameter is constant, and is called $\pi$.

$$\pi = 3{\cdot}142 \text{ to four significant figures.}$$

If the radius of a circle is $r$ inches, then

$$\frac{\text{circumference}}{2r} = \pi;$$

$$\therefore \text{ circumference of circle} = 2\pi r \text{ inches.}$$

**Area of Circle.**—It can be shewn that the area of a circle of $r$ units radius is given by the formula

$$A = \pi r^2.$$

**Sphere.**—A **sphere** is the solid traced out in space, or **generated**, by the rotation of a semicircle about its diameter (Fig. 65). It can be shewn that, for a sphere of radius $r$ inches,

$$A = 4\pi r^2,$$

$$V = \tfrac{4}{3}\pi r^3,$$

where A sq. in. is the surface and V cub. in. is the volume of the sphere.

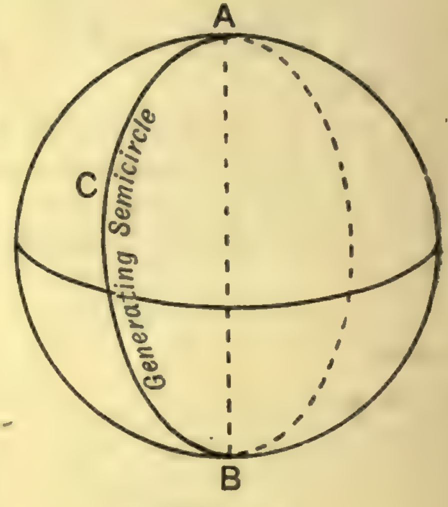


FIG. 65.

**Cylinder.**—A right circular **cylinder** is the solid generated by the rotation of a rectangle about one of its sides (Fig. 66). The side about which the rectangle rotates is the **axis** of the cylinder. The other two sides of the rectangle at right angles to the axis generate the two circular ends.

For a cylinder of height $h$ inches, of which the base radius is $r$ inches,

$$A = 2\pi rh,$$

$$V = \pi r^2 h,$$

where A sq. in. is the curved surface and V cub. in. is the volume of the cylinder.

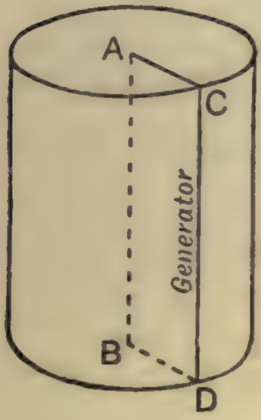


FIG. 66.

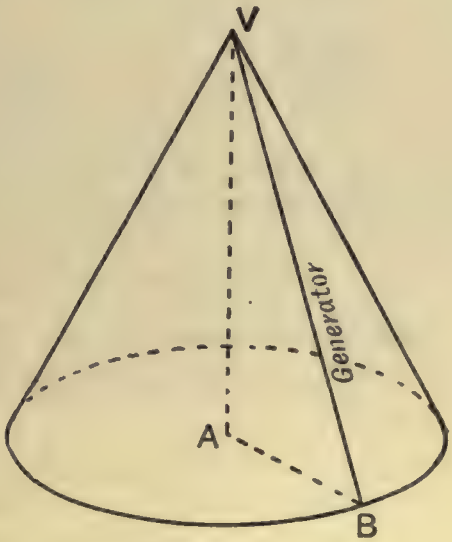


FIG. 67.

**Cone.**—A right circular cone is the solid generated by the rotation of a right-angled triangle about one of the sides containing the right angle (Fig. 67). The side about which the triangle rotates is called the **axis** of the cone; the other side containing the right angle generates the circular base.

For a cone of height $h$ inches, of which the base radius is $r$ inches,

$$l = \sqrt{r^2 + h^2},$$

where $l$ is the length in inches of the slant side (since it is the hypotenuse of the right-angled triangle).

$$A = \pi rl,$$

$$V = \tfrac{1}{3}\pi r^2 h,$$

where A sq. in. is the curved surface of the cone and V cub. in. is its volume.

Ex. 1. *A hollow crank shaft has an external diameter of* 12 *inches and an internal diameter of* 9 *inches. Find its cross-sectional area.*

$$\begin{aligned}
\text{Area of circle 12 in. diameter} &= \pi \times 6^2 \text{ sq. in.}\\
\text{,, \quad 9 \quad ,,} \quad &= \pi \times 4{\cdot}5^2 \text{ sq. in.}\\
\therefore \text{ cross-sectional area of shaft} &= \pi \times 6^2 - \pi \times 4{\cdot}5^2 \text{ sq. in.}\\
&= \pi(6^2 - 4{\cdot}5^2) \text{ sq. in.}\\
&= \pi(6+4{\cdot}5)(6-4{\cdot}5) \text{ sq. in.}\\
&= \pi \times 10{\cdot}5 \times 1{\cdot}5 \text{ sq. in.}\\
&\simeq 3 \times 10 \times 2 \text{ sq. in.}\\
&\simeq 60 \text{ sq. in.}
\end{aligned}$$

$$\begin{array}{r}
31\cancel{4}\cancel{2}\\
105\\ \hline
3142\\
157\\ \hline
329\cancel{9}\\
15\\ \hline
3299\\
1650\\ \hline
5949
\end{array}$$

$$\text{Area} = 59{\cdot}5 \text{ sq. in.}$$

Ex. 2. *A bicycle is going* 10 *miles per hour. How many revolutions do the wheels make per min.? (Diameter of wheels* $=28$ *in.)*

$$\text{Circumference of wheel} = \pi \times 28 \text{ in.}$$

If it makes $n$ revolutions per min., then :

$$\begin{aligned}
&\text{In 1 min. the wheel travels } \pi \times 28 \times n \text{ inches ;}\\
&\therefore \text{ in 1 hr.} \quad \text{,,} \quad \text{,,} \quad \pi \times 28 \times n \times 60 \text{ inches,}\\
&\qquad\qquad\qquad\qquad \textit{i.e.} \quad \frac{\pi \times 28 \times n \times 60}{5280 \times 12} \text{ miles.}
\end{aligned}$$

$$\therefore \frac{\pi \times 28 \times n \times 60}{5280 \times 12} = 10 ;$$

$$\therefore \frac{\pi \times 7 \times n}{88 \times 3} = 10 ;$$

$$\therefore n = \frac{10 \times 88 \times 3}{\pi \times 7}$$

$$\simeq \frac{10 \times 88}{7} \simeq 130.$$

$$\begin{array}{r|r}
 & 1200\\ \hline
2\cancel{1}\cancel{9}\cancel{9} & 2640\\
 & 441\\
 & 1
\end{array}$$

$$n = 120$$

Ex. 3. *A hollow cylinder* L *feet long has an external radius* R *inches and internal radius* r *inches. It is made of material weighing* w *lb. per cub. ft. Find an expression for the weight of the cylinder.*

$$\text{Volume of solid cylinder} = \frac{\pi R^2}{144} \times L \text{ cub. ft.}$$

$$\text{,, \quad hole} \quad = \frac{\pi r^2}{144} \times L \quad \text{,,}$$

$$\therefore \text{ volume of hollow cylinder} = \frac{\pi}{144}\mathsf{L}(\mathsf{R}^2 - \mathsf{r}^2) \text{ cub. ft.}$$

$$\therefore \text{ weight} = \frac{\pi \mathsf{Lw}}{144}(\mathsf{R}^2 - \mathsf{r}^2) \text{ lb.}$$

Ex. 4. *What is the volume of a cone* 6 *in. high, if the base radius is* 2·5 *in.?*

$$\begin{array}{r} 25\phantom{0}\\ 25\\ \hline 50\phantom{0}\\ 125\\ \hline \cancel{6}\cancel{2}\cancel{5}\\ \phantom{00}6284\\ \hline 3750\phantom{00}\\ 125\phantom{00}\\ 50\phantom{00}\\ 2\phantom{00}\\ \hline 3927\phantom{00} \end{array}$$

$$\begin{aligned} \text{Volume} &= \tfrac{1}{3}\pi r^2 h \text{ cub. in.}\\ &= \tfrac{1}{3}\,.\,\pi \times (2\cdot5)^2 \times 6 \text{ cub. in.}\\ &\stackrel{\frown}{=} 36 \text{ cub. in.} \end{aligned}$$

$$\text{Volume} \stackrel{\frown}{=} 39\cdot3 \text{ cub. in.}$$

## EXERCISES XLII.

**1.** Find the area of a circular running track whose outer radius is 140 yards and whose width is 7 yards. ($\pi = 3\frac{1}{7}$.) (M.C.U.)

**2.** Find the volume of a sphere whose radius is 1·5 centimetres, correct to the $\frac{1}{100}$th part of a cubic centimetre. Given that volume of a sphere is $\frac{4}{3}\pi r^3$, where $\pi = 3\cdot1416$ and $r$ is the radius of the sphere. (M.C.U.)

**3.** A sphere of stone is 3 feet in diameter, the weight of the stone per cubic foot is 120 lb. What is the weight of the sphere? (B.E.)

**4.** (*a*) Make a freehand sketch of a triangular prism.

(*b*) If the ends are equilateral triangles, of which each side measures 4 cm. and the length of the prism is 7·5 cm., find the volume of the prism.

(*c*) Make a careful sketch of a piece of paper which would exactly cover the prism. (M.C.U.)

**5.** A carriage wheel of diameter 7 feet makes 60 revolutions per minute. Find the speed of the carriage in feet per second and in miles per hour. ($\pi = 3\frac{1}{7}$.) (M.C.U.)

**6.** (*a*) The diameter of a cylinder is $d$ and the length $l$; what is the volume?

(*b*) Calculate the volume if $d = 7$ cm. and $l = 2\cdot5$ cm. ($\pi = 3\cdot14$.) (M.C.U.)

**7.** Calculate the weight of a metal cylinder of diameter 7 cm., length 10 cm., weight per cubic centimetre 7·5 grams. ($\pi = 3\frac{1}{7}$.) (M.C.U.)

**8.** A circular disc is 24 inches in diameter. A concentric disc is punched out so that the area left is $\frac{2}{3}$ of the original area. Find the radius of the disc removed, to two decimal places. (M.C.U.)

**9.** A metal pipe 50 cm. long has an outside diameter 4 cm. and an inside diameter 2·4 cm. Find (*a*) what volume of water the pipe would hold, (*b*) what would be the weight of the empty pipe if each cubic centimetre of the metal weighed 10 grams. (M.C.U.)

**10.** In Question 9, if the outside diameter of the pipe is 2R cm., the inside diameter $2r$ cm., the length of the pipe $l$ cm., the weight of the metal $w$ grams per c.c. and W the total weight of the pipe, write down an equation connecting these quantities. (M.C.U.)

**11.** Find the weight of a copper pipe of external diameter 5 cm., internal diameter 3·8 cm. and length 25 cm., if one cubic centimetre of copper weighs 8·8 grams. (M.C.U.)

**12.** The weight in grams of a wet drain pipe is given by the formula

$$\pi(r^2 - r_1^2)ld,$$

where $\pi = 3{\cdot}1416$, $r = 24{\cdot}35$, $r_1 = 21{\cdot}74$, $l = 91{\cdot}32$ and $d = 3{\cdot}45$. On drying it loses $\frac{1}{6\cdot3}$rd of its weight. Find its weight when dry, to the nearest dekagram. (M.C.U.)

**13.** Find the value of the following when $R = 12''$, $r = 2''$, $h = 10''$, $\pi = 3\frac{1}{7}$:

(*a*) $\pi(R^2 - r^2)h$. (*b*) $\frac{4}{3}\pi R^3$. (*c*) $2\pi rh$. (M.C.U.)

**14.** A pipe, $\frac{3}{4}''$ internal diameter, is 12 feet long. How many cubic inches of water will it hold? Express this as a fraction of a gallon.

What is the weight of this length of pipe, if its external diameter is 1″, the metal weighing 0·26 lb. per cubic inch? (B.E.)

**15.** A right circular cone has a base 4 in. in diameter and a height of 6 in. What is the slant height, and what is the area of the surface, not including the base? (B.E.)

**16.** The base of a cone is a circle of 3·25 inches diameter. The vertical height is 5·24 inches. It is made of cast iron, which weighs 0·26 lb. per cubic inch. What is the weight of the cone? (B.E.)

**17.** A circular piece of zinc of thickness $t$, diameter D, has a round hole in it of diameter $d$. Find a formula for the volume of the zinc. (B.E.)

**18.** Assuming that a drop of water is spherical and $\frac{1}{10}$ inch in diameter, to what depth will 1000 drops fill a hollow cylinder, whose base has a radius of $\frac{1}{2}$ an inch?

(Vol. of sphere $= \frac{4}{3} \times \frac{22}{7} \times r^3$.) (M C.U.)

# TYPICAL EXAMINATION PAPERS.

## MIDLAND COUNTIES UNION OF EDUCATIONAL INSTITUTIONS.

## PRELIMINARY CERTIFICATE EXAMINATIONS.

### TECHNICAL GROUP. PART I.

### Practical Mathematics.

*Six questions only to be attempted, and Candidates are recommended to try Questions* 1, 2 *and* 3.

*The full working of each sum must be shewn.*

**1.** The sides of a triangle measure 4·1 inches, 5·3 inches and 6·6 inches respectively.

(*a*) Draw the triangle to the scale of half full size.

(*b*) Measure the angles.

(*c*) Find the area of the triangle by any convenient method.

(*d*) On the longest side as base draw an isosceles triangle of equal area.

**2.** If $17{\cdot}63 \times 1{\cdot}022 = 18{\cdot}01786$, write down the values of $1{\cdot}763 \times 10{\cdot}22$ ; $0{\cdot}1763 \times 0{\cdot}1022$ ; $176{\cdot}3 \times 102{\cdot}2$.

**3.** (*a*) The diameter of the base of a cone is 7 cm. and the sloping height is 8 cm. By means of a drawing find the vertical height of the cone.

(*b*) Make a careful drawing of a piece of paper which would exactly cover the curved surface of the cone.

(*c*) Calculate the total surface of the cone.

(*d*) Find the volume of the cone.

$$(\pi = 3\tfrac{1}{7}.)$$

**4.** In a working week of 52 hours a youth earns 16*s.* 3*d.* What should he earn if his week were shortened to 48 hours, but his speed of working increased 10 per cent.?

**5.** Find the value of $x$ in each of the following equations :

(*a*) $2(x-1)-3(2-x)=x+4$ ; (*b*) $\frac{x+5}{4}-\frac{x-4}{5}=1-\frac{x}{20}$ ;

(*c*) $\frac{1}{x}+\frac{2}{x}=1$.

**6.** Find the value of the following when $a=1$, $b=2$, $c=-3$ and $d=4$ :

(i) $(a+b)(b+c)(c+d)$ ; (ii) $a^3+b^3+c^3-3abc$ ;

(iii) $\frac{1}{a^2-b^2}+\frac{1}{a^2+b^2}$ ; (iv) $\frac{1}{(a-b)^2}-\frac{1}{(a+b)^2}$.

**7.** Simplify

(*a*) $(2x+3)(2x-3)=$ (*b*) $(x+8)(x-2)=$

(*c*) $(a+b)(a-b)(a^2+b^2)=$ (*d*) $(x^2+6x-7)\div(x-1)=$

**8.** If 26 is taken from twice a certain number, the answer is 50 ; find the number.

**9.** (*a*) A circular metal washer has a square piece cut out. If the diameter of the washer is 2R, the thickness $t$ and the side of the square $l$, express the volume (V) of the metal by a formula.

(*b*) If $R=5{\cdot}02''$, $t=0{\cdot}19''$, $l=6{\cdot}01''$, find V.

$(\pi=3{\cdot}14.)$

## UNION OF LANCASHIRE AND CHESHIRE INSTITUTES.

### PRELIMINARY TECHNICAL COURSE (FIRST YEAR).

### Practical Mathematics.

*You may attempt not more than* SIX *of the questions in this subject.*

*In making calculations, shew the steps of your work, as value will be attached to a wrong answer if the principle on which it is worked out is good.*

*Questions marked * have accompanying diagrams.*

*Drawings and sketches must be made in pencil.*

**1.** Find the value of

(*a*) $\frac{2\frac{1}{3}\times5\frac{6}{7}}{1\frac{5}{9}}$. (*b*) $\frac{3{\cdot}75\times2{\cdot}5}{0{\cdot}3125}$.

**2.** Find the total length represented by $\frac{1}{2}'' + \frac{5}{8}'' + \frac{3}{16}'' + \frac{9}{32}$.

**3.** By what number must 3·65 be multiplied to give (*a*) 365, (*b*) 3650, (*c*) 0·365?

***4.** By means of the given scale (see Fig. 68) state the length of each of the four given lines A, B, C, D.

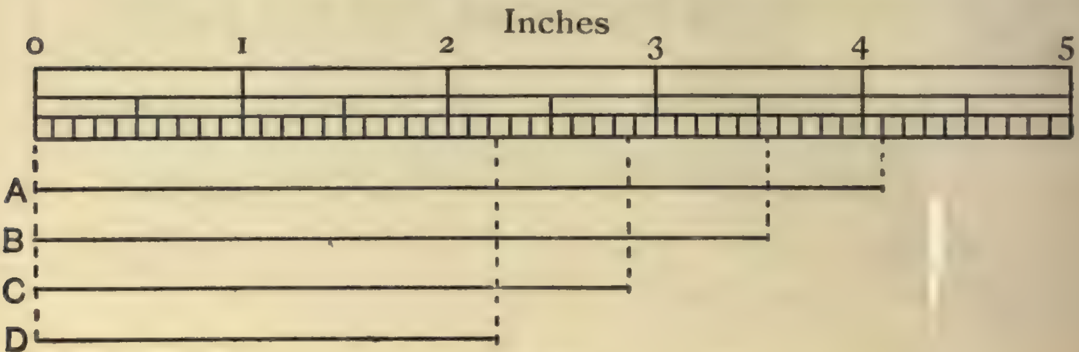


FIG. 68.

On the squared paper draw a square whose perimeter $= A + B + C + D$, and calculate its area.

**5.** A line of 6 in. is divided into ten equal parts, and 1, 3 and 7 parts are taken to form three other lines. Write down the length of each of these lines.

**6.** Find $x$ for the equation (*a*) $x^2 = 3^2 + 4^2$, (*b*) $x^2 = 13^2 - 5^2$.

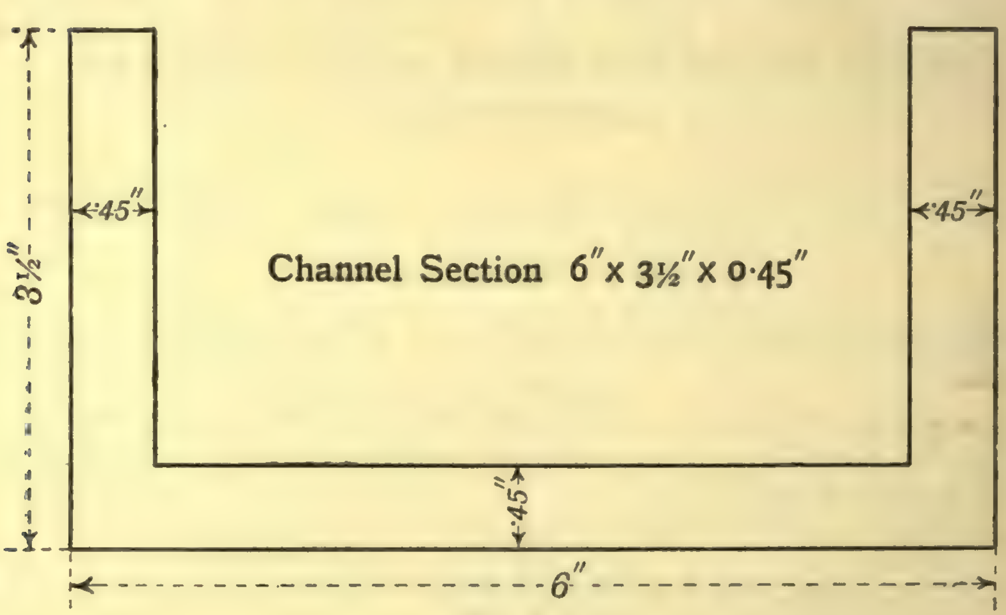


FIG. 69.

***7.** The given figure (see Fig. 69) represents approximately the section of a rolled channel bar.

Draw the section on the squared paper, full size, and calculate its area in square inches.

NOTE.—*When drawing to scale on the squared paper you must work to the lines on the paper and not use an ordinary scale.*

**8.** Two numbers differ by 7, and their sum is 15. Find the numbers.

**9.** Find the values of the following expressions:

(1) $ab+bc-ca$, (2) $a^2c-2b^2+c^2$,

when $a=2$, $b=3$, $c=4$.

**10.** A rectangular tank, 8 ft. long, 6 ft. wide, contains 240 cubic feet of water, and 40 cubic feet of water are drawn off. Find, in feet and inches, the height of the water remaining in the tank, and the amount by which the level of the water has fallen.

## UNION OF LANCASHIRE AND CHESHIRE INSTITUTES.

### PRELIMINARY TECHNICAL COURSE (SECOND YEAR).

### Practical Mathematics.

*You may attempt not more than* SIX *of the questions in this subject.*

*In making calculations, shew the steps of your work, as value will be attached to a wrong answer if the principle on which it is worked out is good.*

*Questions marked * have accompanying diagrams.*

*Drawings and sketches must be made in pencil.*

**1.** The exact value of a quantity is 26·827. What is its approximate value

(*a*) to the nearest whole number,
(*b*) „ „ tenth,
(*c*) „ „ hundredth?

What is the percentage error—to two places of decimals—when the approximate value (*a*) is taken?

**2.** The circumference of a circle is 54·98 in. and its diameter is 17·5 in.; calculate the value of $\pi$ to three decimal places.

**3.** Find the numerical value of $\tau$ in the following expression:

$$\tau=\frac{\pi d^3 f}{16},$$

where $\pi=\frac{22}{7}$, $d=5$, $f=8000$.

**4.** A ladder 25 ft. long stands upright against a wall. Find how far the bottom of the ladder must be pulled out from the wall so as to lower the top 1 ft.

On the squared paper draw the figure to a *scale of* 1 *in.* = 5 *ft.*, and verify your calculation.

**5.** Write down the value of $(x+y)^2$, $(x-y)^2$, $(a-2b)^2$, and also the factors of $x^4-16y^4$ and $a^2-7ab+12b^2$.

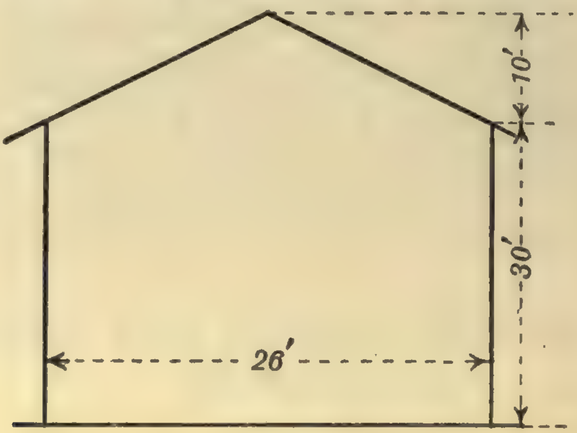


FIG. 70.

***6.** The dimensioned sketch (see Fig. 70) represents the gable end of a house. Find its area in square feet.

**7.** A rectangle has a length represented by $(2x+y)$ and a height $(x-2y)$. Write down the expression for the area of the rectangle.

Shew that your result is correct by putting $x=2$, $y=\frac{1}{2}$, in $(2x+y)(x-2y)$ and the expression you obtain for the area.

**8.** Solve (*a*) $(x-1)(x-2)-(x-2)(x-3)=6$;

(*b*) $2(x-3)+3(x-4)=2$.

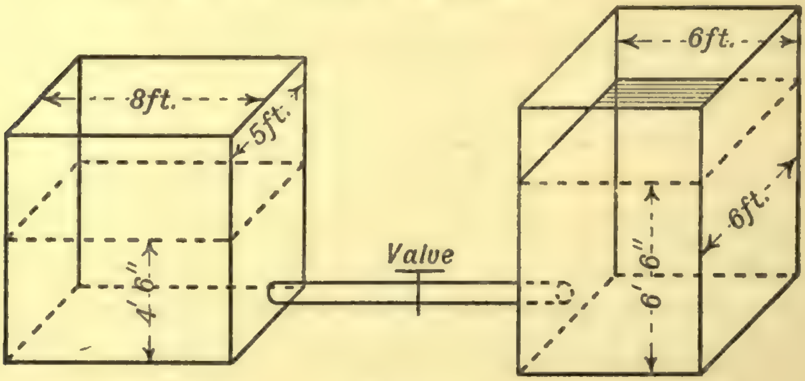


FIG. 71.

***9.** Two rectangular tanks, standing on a level foundation, are connected by a horizontal pipe with valve (see Fig. 71), and when the valve is closed the tanks contain water to the given heights.

When the valve is opened, water will flow from one tank to the other until the water stands at the same level in the two.

Calculate the height of the water in both tanks when the valve is open.

***10.** In an experiment on a spiral spring it is found that a pull of 10 lb. stretches the spring 1·5 in.

On the squared paper draw a *straight-line graph shewing the relation between pull and stretch* for any value of the pull up to 40 lb.

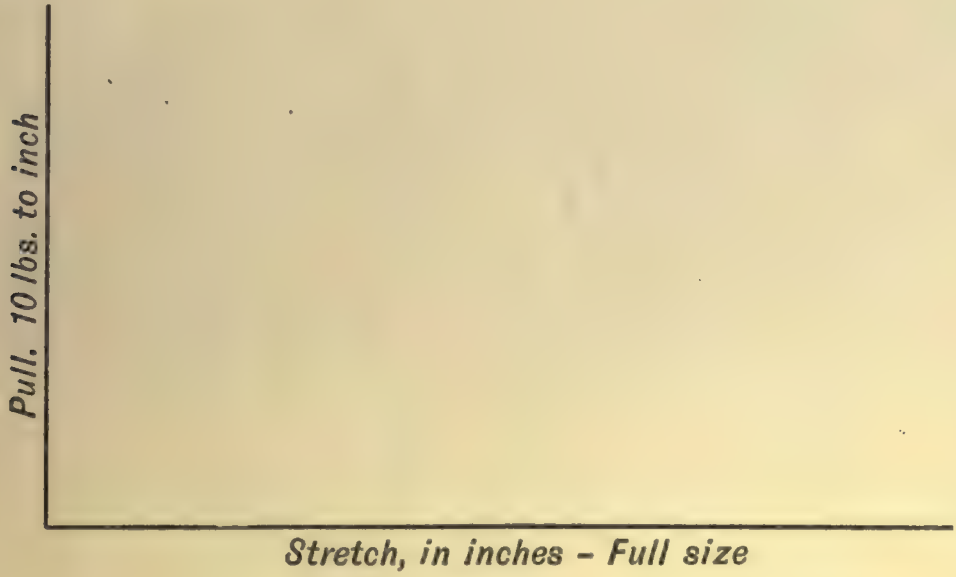


FIG. 72.

Plot *stretch* horizontally, full size, and *pull* vertically to a scale of 10 lb. to the inch (see Fig. 72).

From the graph find the stretch produced by a pull of 21 lb.

# ANSWERS TO EXERCISES.

## CHAPTER I.

### Exercises I.

**1.** £7. 19*s*. $7\frac{1}{2}$*a*. **2.** 12 c. + 4 h. + 100 s. + 30 p.; 146.
**3.** $11\frac{1}{2}$. **4.** $w+p+\mathsf{W}$.
**5.** $r_1+\mathsf{R}+r_2+\mathsf{R}'$; $30\frac{3}{4}$. **6.** $\mathsf{W}+\mathsf{W}_1+\mathsf{W}_2+\mathsf{W}_3$ lb.
**7.** (*a*) $6\frac{1}{2}$; (*b*) $83\frac{1}{2}$; (*c*) $16\frac{3}{4}$. **8.** (*a*) 56; (*b*) 21; (*c*) $42\frac{1}{2}$.
**9.** $8x+2y+2z$. **10.** $9x+5y+9z$.
**11.** $2\mathsf{L}+2\mathsf{B}$ yd.; (*a*) $2\mathsf{H}+2\mathsf{L}$ yd.; (*b*) $2\mathsf{H}+2\mathsf{B}$ yd.

### Exercises II.

**1, 2** and **3.** Negative. **4.** (*a*) 102°; (*b*) 22°; (*c*) $-38°$.
**5.** $(x+32)°$, $32-t$ degrees. **6.** (*a*) 140°; (*b*) 252°; (*c*) $(212-t)°$.
**7.** $-273°$, $373°$, $-273°$. **8.** $(273+t)°$.
**9.** $s$ ft. upwards; 800 ft. above sea level; 200 B.C.; 400 A.D.; 40 ft. per sec. S.E.; 50 miles per hr. E.
**10.** $-400$ yd. high; $-950$ ft. below sea level; $-820$ ft. above sea level; $-y$ ft. South; $-v$ ft. per sec. W.; $-200$ A.D.; $-100°$ below zero; $-57°$ above zero.
**11.** £3. 15*s*. 8*d*. **12.** $-2$. **13.** $-\frac{5}{4}$.
**14.** $-4$. **15.** 5 cows. **16.** $-1$ pound.
**17.** 27 *a*'s. **18.** $-3$ *a*'s. **19.** $-8x$.
**20.** $2y$. **21.** $p-a$ feet. **22.** $\mathsf{R}-r_1-r_2$ ohms.
**23.** $\mathsf{W}-w$ lb. **24.** $\mathsf{W}_1+\mathsf{W}_2-\mathsf{W}$ lb.

### Exercises III.

**1.** $a$ times the sum of $b$ and $c$; $x$ times the result of subtracting $z$ from 1; add $p$ to 3 and multiply their sum by the result of subtracting 5 from $y$.
**2.** $np$ inches. **3.** $wl$ lb. **4.** $nl$ centimetres.
**5.** $200\times3\frac{1}{10}\times r$ ohms; *i.e.* $620r$ ohms. **6.** $wln$ lb.
**7.** 750 feet or 250 yd. **8.** $13x+30+w$ inches.
**9.** $9n$ inches. **10.** Length 91 ft.; breadth 5 ft. $6\frac{1}{2}$ in.
**11.** $5\mathsf{W}+w$ lb.; 185 lb. on each. **12.** $7nv$ cub. ft.

## Exercises IV.

**1.** 66; £1 for expenses. **2.** 64 yd.

**3.** 7 layers; 2″ space. **4.** $12l$ inches, $\frac{12l}{9}$ or $\frac{4l}{3}$.

**5.** 9. **6.** 12 strips; 2″. **7.** 1864. **8.** £$x-na$.

**9.** $\frac{D}{p}+1$. **10.** 14 times; $w$ gal.

**11.** 120 times. **12.** $\frac{12L-x-na}{n+1}$ inches.

## Exercises V.

**1.** $\because\ 5>3\frac{3}{4},\ \therefore\ 2+5>7\frac{1}{2}$. **2.** $\angle P \not> \angle Q$.

**3.** $\frac{10}{5}>\frac{10}{6},\ \because\ 5<6$. **4.** If $P>Q,\ \therefore\ P+x>Q+x$.

**5.** $\because\ R<S,\ \therefore\ R-a<S-a$. **6.** $\because\ x=a,\ \therefore\ 2x=2a$ and $2\frac{1}{2}x>2a$.

**7.** If $y>z,\ \therefore\ ay>az$, if $a$ is $+^{ve}$.

## Exercises VI.

**1.** $s$ equals the product of $v$ and $t$; P equals the product of $m$ and $f$; the result of subtracting $u$ from $v$ equals the product of $f$ and $t$; add $r$ to R and divide E by their sum, the result equals C; the product of $p$ and $v$ is unity; to $l$ add the quotient of $m$ divided by $n$; subtract $y$ from $x$, divide the remainder by $r$, and into $r$, add the quotients.

**2.** If $p$ is greater than $q$, then $mp$ is greater than $mq$ as long as $m$ is positive.

**3.** (*a*) $y-4$; (*b*) $\frac{x+y}{z}$; (*c*) $3x+14-4$ or $3x+10$; (*d*) $3x+6$.

**4.** $x+1,\ x-1$. **5.** $a+2,\ a-2$. **6.** $x-7$ years, $x+a$ years.

**7.** $20x-y$ shillings. **8.** $m-24$ miles. **9.** $W+725$ lb.

**10.** $\frac{W}{A}$ lb. per sq. in. **11.** $\frac{P}{A}$ lb. per sq. in.

# CHAPTER II.

## Exercises VII.

**1.** Five hundred and seventy-eight, three tenths and two hundredths; fifty-seven, eight tenths, three hundredths and two thousandths; five tenths, seven hundredths, eight thousandths and three ten-thousandths; five thousandths and seven ten-thousandths; five million, seven hundred and eighty-three thousand.

**2.** 70825·2, 396·405, 7·041, 600·02, 40·004, 90·097, 798·401.

**3.** 2002, 0·01732, 0·000045, 578000000, 0·000578, 314, 7004, 0·000008888.

**4.** 1·2047. **5.** 0·0815. **6.** 35·6, 1·97, 12, 0·172.

**7.** 8000 miles. **8.** 93000000 miles. **9.** 0·00002 inch.

**10.** 1 cubic cm. is 0·000001 cubic m. and 0·001 litre.

## Exercises VIII.

**1.** $1{\cdot}007\times10^{2}$, $4{\cdot}317\times10^{1}$, $2{\cdot}0005\times10^{1}$, $4{\cdot}73\times10^{5}$, $9{\cdot}9901\times10^{2}$, $1{\cdot}000001\times10^{4}$, $8{\cdot}763\times10^{-1}$, $8{\cdot}0073\times10^{-1}$, $8{\cdot}32\times10^{-4}$, $8{\cdot}007\times10^{-3}$, $8{\cdot}007\times10^{+2}$.

**2.** (*a*) $8\times10^{3}$ miles ; (*b*) $2{\cdot}9\times10^{4}$ ft. ; (*c*) $9{\cdot}3\times10^{6}$ miles ;
(*d*) $2{\cdot}75\times10^{5}$ times as far ; (*e*) 0·000002 inch ;
(*f*) $3{\cdot}0\times10^{5}$ km. per sec. (*g*) $2\times10^{3}$ ft. per sec.
(*h*) $2{\cdot}6\times10^{1}$ lb. (*i*) $7\times10^{5}$ horse-power, $2{\cdot}75\times10^{4}$ tons ;
(*j*) 1 cubic metre $=10^{6}$ c.c.
(*k*) 1 c.c. $=10^{3}$ c.mm. and $1{\cdot}0\times10^{-3}$ litre.

## Exercises IX.

**1.** (1) $10^{3}$; (2) $10^{-6}$; (3) $10^{5}$; (4) $10^{1}$; (5) $10^{-5}$.

**2.** (1) $7{\cdot}8\times10^{-2}$; (2) $5{\cdot}64\times10^{1}$; (3) $7{\cdot}2\times10^{2}$; (4) $4\times10^{3}$; (5) $3{\cdot}978\times10^{-2}$.

**3.** (1) $10^{3}$; (2) $1{\cdot}6\times10^{3}$; (3) $3{\cdot}604\times10^{-1}$; (4) $2{\cdot}448\times10^{1}$; (5) $1{\cdot}108\times10^{-3}$.

## Exercises X.

**1.** (*a*) 5·042 m., 2004 m., 4504 m. ; (*b*) 0·005042 Km., 2·004 Km., 0·004504 Km. ; (*c*) 504·2 cm., 200400 cm., 450·4 cm.

**2.** 8 m. 8 dm. 1 cm. 9 mm. or 8·819 m.

**3.** 1245 cm. **4.** 450 cm.

**5.** (*a*) 5085 mg., 24084000 mg., 7045024 mg. ; (*b*) 0·005085 Kg., 24·084 Kg., 7·045024 Kg. ; (*c*) 5·085 gm., 24084 gm., 7045·024 gm.

**6.** 46 Kg. 160 gm. 30 mg. or 46·16003 Kg. **7.** 3998·89 m.

**8.** $5\times10^{4}$ sq. cm. **9.** 1 sq. m. $=10^{4}$ sq. cm. $=10^{6}$ sq. mm.
1 sq. cm. $=10^{2}$ sq. mm.

**10.** $10^{6}$ c.c., $10^{15}$ c.c.

**11.** 9 times. **12.** $10^{6}$ mg.

**13.** (*a*) $10^{-6}$; (*b*) $10^{3}$; (*c*) $10^{-4}$; (*d*) $10^{3}$; (*e*) $10^{6}$.

## Exercises XI.

**1.** 23 c.c. **2.** 17·6 gm., 1000 gm. **3.** 50 c.c.

**4.** 13·620 gm. per c.c. **5.** 930 gm. **6.** 1360 gm.

**7.** $W/v$ gm. per c.c. **8.** $V\rho$ gm. **9.** $vs$ gm.

**10.** 2·86 lb. **11.** 850 lb., 168 lb. **12.** 1033·6 gm.

## CHAPTER III.

### Exercises XII.

**1.** (*a*) 1; (*b*) 3; (*c*) 4; (*d*) 2; (*e*) 4; (*f*) 3; (*g*) 4; (*h*) 4; (*k*) 2; (*l*) 2.

**2.** (*a*) $8 \times 10^3$; (*b*) $3{\cdot}44 \times 10^2$; (*c*) $1{\cdot}293 \times 10^0$; (*d*) $1{\cdot}1 \times 10^{-5}$;
(*e*) $3{\cdot}937 \times 10^{-1}$; (*f*) $2{\cdot}54 \times 10^0$; (*g*) $4{\cdot}534 \times 10^2$;
(*h*) $2{\cdot}205 \times 10^0$; (*k*) $7{\cdot}8 \times 10^0$; (*l*) $2{\cdot}6 \times 10^0$.

**3.** (*a*) 587·0, 93·7, 20·2, 20·1, 0·00872, 877000, 1·00, 4440, 401·0, 0·920; (*b*) 590, 94, 20, 20, 0·0087, 880000, 1·0, 4400, 400, 0·92; (*c*) 600, 90, 20, 20, 0·009, 900000, 1·0, 4000, 400, 0·9.

**4.** (*a*) 6729 m.; (*b*) 6730 m.; (*c*) 7 Km.

**5.** (*a*) Acreage = 32,580,000, population = 30,810,000;
(*b*) acreage = 32,578,000, population = 30,811,000;
(*c*) acreage = 32,600,000, population = 30,800,000.

**6.** (*a*) £224,800,000; (*b*) £224,846,000; (*c*) £224,800,000.

### Exercises XIII.

**1.** 181100. **2.** 59·67. **3.** 838200. **4.** 19640.
**5.** 886·6. **6.** 0·09156. **7.** 9 ft. 7⅛ in.
**8.** (i) 2 ft. 7½ in.; (ii) 3 ft. ⅛ in.: (iii) 2 ft. 1¾ in.
**9.** (i) 3 ft. 1⅜ in.; (ii) 1 ft. 9⅝ in.

### Exercises XIV.

**1.** 16. **2.** 6·411. **3.** £98. 17*s*. 10½*d*. **4.** £19. 10*s*. 10½*d*.
**5.** 8·164. **6.** 10. **7.** 9·6. **8.** 1998 m.
**9.** (*a*) 54·24 gm.; (*b*) 0·1 Kg. **10.** 449. **11.** 495. **12.** 787·2.

### Exercises XV.

**1.** $3p - q + 2r$. **2.** $9x - 5y + 6z$. **3.** $4x + z$. **4.** $6a - 0{\cdot}9b$.
**5.** $5{\cdot}7a + 5{\cdot}1b - 6{\cdot}6c$. **6.** $2a + b$. **7.** $2y - z$. **8.** $-x - 2y + 3z$.
**9.** $0{\cdot}6a - 1{\cdot}5b + 1{\cdot}3c$. **10.** (i) $-12$; (ii) 13; (iii) $-50$.

### Exercises XVI.

**1.** 7·49. **2.** 23·4. **3.** 234. **4.** 864. **5.** 1752.
**6.** 1715000. **7.** 115,000,000. **8.** 34750. **9.** 4290.
**10.** 0·02316. **11.** 0·007988. **12.** 28·26.
**13.** 219·71 sq. ft., 219·24 sq. ft., 220 sq. ft.
**14.** 609·0. **15.** 4510000.

### Exercises XVII.

**1.** 22·7. **2.** 153. **3.** 0·0000153. **4.** 0·00000188. **5.** 0·000107.
**6.** 11. **7.** 149000. **8.** 442·2. **9.** 0·001868. **10.** 0·08775.
**11.** 0·007828. **12.** 3·498. **13.** 2·54. **14.** 17·8 ft. per sec.
**15.** 15·4. **16.** 746 watts = 1 H.P., 1 watt = 0·00134 H.P.

### Exercises XVIII.

**1.** $a \times a \times a \times a$, $y \times y \times y \times y \times y$. **3.** $b^9$, $c^3$, $y^9$, $z^3$.

**4.** $x^3 = 8$, 429, 3180; $\frac{1}{x^2} = 0{\cdot}25$, 0·0816, 0·00463.

**5.** 16·83. **6.** (*a*) 0; (*b*) 39·80.

### Exercises XIX.

**1.** 0·16, 2·25, 640,000, 22,500, 0·0361, 0·0004, 0·00000121, 4900, 2·89, 0·000256, 324,000,000, 0·000000000196.

**2.** 1·3, 1·7, 160, 1·6, 0·16, 1·1, 0·4, 0·3, 0·01, 1·5, 0·015, 0·013, 0·0018.

**3.** 1616·895. **4.** 6·14. **6.** 17, 1.

**7.** (14, 15), (1·3, 1·4), (0·08, 0·09), (15, 16), (50, 60), (0·3, 0·4), (6, 7), (60, 70), (1·9, 2·0), (0·19, 0·20), (3, 4), (200, 300).

**8.** 26, 3·1, 550, 195, 0·66, 0·126.

**9.** 6·56, 0·548, 1·73, 0·0237, 1·77, 1·78. **10.** 8·4853. **11.** 0·3507136.

## CHAPTER IV.

### Exercises XX.

**3.** $10^4$. **4.** $10^2$. **5.** $1760^2 = 3097600$.

### Exercises XXI.

**2.** (i) 3·0 sq. in.; (ii) 7·8 sq. in.; (iii) 96 sq. in.

**3.** 40 cm. **4.** 9·0 in. **5.** $\frac{A}{a}$ cm. **6.** 16.

**7.** 31·25 sq. in. Actual area $216^2$ times as large.

**8.** 2·71 sq. in. **9.** 3·90 sq. cm. **10.** 46500 sq. yd.

**13.** $\perp^{r} = 2{\cdot}0$ in.; area 2·80 sq. in. **14.** 52° each. **16.** 4·25 sq. in.

**17.** 21·5 sq. in., 550 acres.

### Miscellaneous Exercises. (p. 62.)

**1.** 103 ft., 60 ft. **2.** 18·8 cm. **3.** Two. Yes; only one.
**4.** Yes. **5.** 908 sq. ft. **6.** 4·25 sq. in.
**7.** 25 sq. in. **8.** 45 sq. ft. **9.** 5·43 sq. in.
**10.** Base, 3·73 in.; height, 2·76 in.; 5·15 sq. in. **11.** 6·41 sq. in.

**12.** 5·80 sq. in. **13.** 6·70 sq. in. **14.** 2·5 sq. in.
**15.** (*c*) 0·90 sq. in.; (*d*) 0·95 in. **16.** 2·48 sq. in., 1·58 in.
**17.** 670 sq. yd. **18.** 14·1 sq. cm. **19.** 1·92 sq. in.

### Exercises XXIV.

**1.** 72. **2.** *abc* cubic units, 44·2 c. in., 17 sq. in. **3.** 135.
**4.** 158 cub. ft., 23700 lb. **5.** 40·4 sq. cm., 766 gm. **6.** 4·8 cm.
**7.** 720 gm. **8.** 211200 c. ft. **9.** 189·6 gall. **10.** 720 gm.
**11.** 72·637 c. in., 74·291 c. in., 73·464 c. in., 1·125 %.
**12.** 170100 c. ft. **13.** 18750 c. ft. **14.** 11 sq. ft., 49·5 c. in., 203 lb.
**15.** 13·36 in. **16.** 102·5 sq. ft. **17.** 0·7 cm.
**18.** 1995 c.c.; 7 cm., 19 cm., 15 cm.

## CHAPTER V.

### Exercises XXV.

**1.** $t^2=\frac{4\pi^2 l}{32{\cdot}2}$. **2.** Work = product of load and distance it is raised.

**3.** Available horse-power is product of the quantity of water in gallons per second, the distance fallen and 0·0182.

**4.** $\frac{1}{x}+\frac{1}{y}=\frac{x+y}{xy}$. **5.** $\rho=\frac{\mathrm{W}}{\mathrm{V}}$. **6.** Volume $=\frac{w}{\rho}$. **7.** 4.

**8.** $\frac{\mathrm{W}}{\mathrm{W}-w}$. **9.** $\frac{\mathrm{F}-32}{9}=\frac{\mathrm{C}}{5}$. **10.** $s=1{\cdot}42\sqrt{h}$.

**11.** Distance of horizon in nautical miles is approximately equal to 1·23 times the square root of the height in feet of the eye of the observer.

**12.** Dryness fraction is the quotient (weight of dry steam ÷ total weight of steam).

### Exercises XXVI.

**1.** 120°. **2.** 10°. **3.** (*a*) 52·5°; (*b*) 90°. **4.** 60°, 30°.
**5.** 18 cm., 45 cm. **6.** (*a*) 0·5 sec.; (*b*) 64 ft.
**7.** (*a*) 20 lb. per sq. in.; (*b*) 400 lb. per sq. in.; (*c*) 8 cub. ft.; (*d*) 1·0 cub. ft.

### Exercises XXVII.

**1.** 5. **2.** −3. **3.** −0·54. **4.** $\frac{1}{8}$. **5.** 2. **6.** 4. **7.** 6.
**8.** 2. **9.** 5 and 15. **10.** 30·1 and 15·6. **11.** 45·6 and 30·4.
**12.** 6″ and 2″. **13.** £10 B, £20 A, £30 C. **14.** £20.
**15.** 81 and 14·5. **16.** Each man £$\frac{20}{3}$, woman £$\frac{40}{9}$, child £$\frac{20}{9}$.

## Exercises XXVIII.

**1.** Three times the sum of 5 and 2; the difference of 4 and 2, multiplied by the sum of 3 and 5; subtract 2 from 4 and multiply by the result of subtracting 5 from 3; to the sum of 7 and 2 add the difference of 5 and 3; subtract 7 from the sum of 5 and 4 and from the result take the sum of 4 and 6; **21, 16, −4, 11, −8.**

**2.** (i) 0·2; (ii) 0·2; (iii) 0·0977; (iv) 3·67. **3.** (i) 15·4; (ii) 57·9.

**6.** (i) $x-y+z$; (ii) $2x-3y-3z$; (iii) $p+5q$; (iv) $x$; (v) $2x+5y+18$; (vi) $3y+z+2x-14$; (vii) $15-2z+4y-x$; (viii) $-z-3x+y$.

## Miscellaneous Exercises. (p. 83.)

**1.** 50. **2.** 23025·1, 2·3025.

**3.** H.P. 34·65″, I.P. 50·79″, L.P. 57·48″, 39·37″ stroke, crank shaft dia. 16·14″, $3{\cdot}86 \times 10^6$ sq. in. heating surface, 199 lb. per sq. in.

**4.** 86·875. **5.** 2. **6.** 17·22. **7.** $8\frac{1}{2}''$, 0·0083 ft. **8.** 908 sq. ft.

**9.** 2, 49. **10.** (*a*) 42·5; (*b*) 70; (*c*) 0·053; (*d*) 0·7024.

**11.** 20. **12.** 3548 mm. **13.** 219 ft. 11 in. **14.** 28·2 sq. cm.

**15.** (*a*) 4; (*b*) $\frac{2}{3}$. **16.** 0·375′ or $4\frac{1}{2}''$. **17.** 8. **18.** (*a*) 3; (*b*) 4.

**19.** 1865·23, 1·4084, 0·03892, 25·697. **20.** 1127″.

**21.** 5·6 cm., 8·4 cm., 47·04 sq. cm. **22.** 1·975″, 3·725″.

**23.** 540 revs., 5400.

## CHAPTER VI.

## Exercises XXIX.

**2.** In 1857: E. 19·2 millions, S. 2·98 millions, I. 6·04 millions.
In 1897: E. 31·1 ,, S. 4·28 ,, I. 4·54 ,,
In 1894. In 1875. In 1885. Population of Ireland began to decrease about 1843. Less rapidly.
England: now 0·36 million per ann.; in 1851, 0·20 million per ann.
Scotland: now 0·030 ,, ,, ; in 1851, 0·025 ,, ,,

**4.** At 48, 35·2 yrs., 24·4 yrs.

**5.** (i) Dec. 30th; (ii) Dec. 15th; (iii) Dec. 22nd.

**6.** 6·25, 10·24, 2·89, 3·16, 4·47. **7.** 11/8, 34/1.

**8.** +1 Kg., 0·0 Kg., −2·0 Kg., 200 Kg.

**9.** Decrease 1st two weeks, then increase. All three refer to iron. Price steady: steady demand and constant production. Tin. £13. 5*s.* 0*d.* per month tin; 15*s.* per month copper; £2 per month antimony; 17*s.* per month spelter; £1. 18*s.* 0*d.* per month lead; £3. 13*s.* 0*d.* per month haematite, £4. 2*s.* 0*d.* Scotch, £4. 10*s.* 0*d.* Cleveland.

**10.** 4·5. **11.** 12·32 in., 13·5 oz.

**12.** Feb. 7·3, June 12·2, Oct. 10·6; August, March; 1·70 litres per m. per month.

**13.** A, 17·2 C.P.; during 1st 100 hrs.; 15·3 C.P.; B and C do not burn steadily at first.

**14.** 0·0065 in. per lb. up to elastic limit; 5·3 tons per sq. in., 15·9 tons per sq. in.

**15.** B, 1·95; A, 0·78; 0·82, 1·03, 1·20; A, 11·1 sq. cm., 9 watts; B, 25 sq. cm., 7·8 watts.

**16.** 552°.

## CHAPTER VII.

### Exercises XXX.

**1.** $3 \times 2^3$, $3 \times 5^2$, $5 \times 23$, $5^2 \times 3^2$, $2^2 \times 191$.

**2.** $a, b, c$; $2, a, x$; $6, p, (r+s)$.

**3.** $\frac{1}{2}, t, (v+u)$; $\frac{1}{2}, n, \{2a+(n-1)d\}$; $(v-u), (v+u), \frac{1}{2}, \frac{1}{f}$;

$P,\ 1+\frac{r}{100},\ 1+\frac{r}{100}$; $(a+b), (c+d)$; $1,\ a+b(c+d)$.

**4.** (i) $a$; (ii) $ab$; (iii) $p+q$; (iv) $5(y+z)^2$.

**5.** (i) $3$; (ii) $7 \times 2^2$; (iii) $5 \times 3$; (iv) $2 \times 3^2$.

**6.** (i) $a$; (ii) $3p$; (iii) $6x^2$; (iv) $3abc$.

### Exercises XXXI.

**1.** (i) 300; (ii) 600; (iii) $30ab$; (iv) $abc$; (v) $12a^2b$; (vi) $36a^2bcx$; (vii) $12abc(x+y)^2(p+q)$.

**2.** H.C.F.'s (i) $ab$; (ii) $ae(l+m)$;
L.C.M.'s (i) $a^2b^2c(d+x)^2$; (ii) $a^2be^3(l+m)^2(m+n)$. **3.** 112.

**4.** (i) $\frac{3}{4}$; (ii) $\frac{7}{24}$; (iii) $\frac{b-a}{ab}$; (iv) $\frac{yz+xz-xy}{xyz}$.

(v) $\frac{R_2R_3R_4+R_1R_3R_4+R_1R_2R_4+R_1R_2R_3}{R_1R_2R_3R_4}$; (vi) $\frac{7}{12a}$;

(vii) $\frac{2x}{3}$; (viii) $\frac{13}{5yz}$; (ix) $\frac{2}{3x}$; (x) $\frac{6bc+4ac-3ab}{12abc}$;

(xi) $\frac{8a-3c}{6b}$; (xii) $\frac{6c-4c^2+3ab}{a^2bc^2}$.

### Exercises XXXII.

**1.** $\frac{3}{4}$. **2.** $\frac{96}{9}$. **3.** $\frac{7}{5}$. **4.** $\frac{40}{21}$.
**5.** $\frac{8}{9}$. **6.** $\frac{8}{77}$. **7.** $\frac{14}{9}$. **8.** $\frac{156}{205}$.
**9.** $\frac{3}{4}$. **10.** $\frac{1}{5}$. **11.** $\frac{1}{3}$. **12.** $\frac{1}{6}$.
**13.** $\frac{6}{7}$. **14.** $\frac{1}{4}$. **15.** $\frac{1}{15}$. **16.** $\frac{c}{a}$.
**17.** $\frac{c}{d(x+y)}$. **18.** $\frac{x-y}{(p+q)(x+y)}$. **19.** 1. **20.** 1. **21.** $\frac{x}{v}$.

## Exercises XXXIII.

**1.** $\frac{7}{6}$, $\frac{4}{3}$, $\frac{1}{3}$, $\frac{51}{50}$, $\frac{3}{2}$; 1·167, 1·333, 0·3333, 1·02, 1·5.

**2.** $\frac{16}{24}$, $\frac{9}{24}$, $\frac{20}{24}$, $\frac{22}{24}$. **3.** $\frac{xyz(p+q)^2}{y^2z^2(p+q)^2}$, $\frac{abz^2(p+q)}{y^2z^2(p+q)}$, $\frac{(a+b)y}{y^2z^2(p+q)^2}$.

**4.** 10, 16, 8, 15, 16, $ac$, $x(p+q)$, $\frac{(a+b)z}{c}$. **5.** $\frac{3}{5}$, $\frac{1}{3}$, $\frac{2x}{y}$, $\frac{2x}{3}$.

**6.** $\frac{5}{128}$ or 0·03906; $\frac{123}{128}$ or 0·96094.

**7.** $\frac{1}{24}$th, $\frac{1}{8}$th, $\frac{1}{6}$th, $\frac{1}{3}$rd. **8.** $\frac{1}{12}$, $\frac{1}{16}$; $\frac{41}{48}$.

**10.** (i) $\frac{x-2}{x(x+3)}$; (ii) $\frac{a(p+q)}{12b(p-q)}$; (iii) $\frac{(x-2)(x+1)}{2x+1}$;
(iv) $\frac{a(a+b)}{x^4y}$; (v) $\frac{(l-m)(l+m)(a+b)}{a(a-b)}$; (vi) $\frac{6(x-y)(a+b)}{71pqr(x-y+a+b)}$.

## Exercises XXXIV.

**1.** (i) $\frac{7}{12}$; (ii) $15\frac{7}{30}$; (iii) $6\frac{7}{30}$; (iv) $\frac{53}{105}$; (v) $1\frac{1}{8}$; (vi) $\frac{7}{8}$;
(vii) $-1\frac{1}{56}$; (viii) $-1\frac{5}{56}$.

**2.** $\frac{24}{64}+\frac{20}{64}+\frac{14}{64}+\frac{3}{64}=\frac{61}{64}$. **3.** $\frac{ax^2+4x^3+3+bx}{x^4}$.

**4.** (i) $\frac{3x+2}{6x}$; (ii) $\frac{bc+ac-ab}{abc}$; (iii) $\frac{yx-y}{x}$;
(iv) $\frac{y-3x}{6}$; (v) $\frac{3z+y-2x}{xyz}$; (vi) $\frac{b}{a^2}$;
(vii) $\frac{3a(a+b-c)+2(a-b+c)}{3a^2}$; (viii) $\frac{4ax}{a^2-x^2}$;

**5.** (i) $-\frac{9}{16}$; (ii) $\frac{1}{2}$; (iii) $\frac{172}{441}$; (iv) $\frac{9c}{4d}$; (v) $\frac{1}{6}$; (vi) $\frac{a^2}{b^2}$. **6.** $\frac{184}{639}$.

**7.** $\frac{8xv}{y^2-4x^2}$. **8.** (i) $\frac{x(a+c)-abc}{a(a-b)}$, 0; (ii) $\frac{(ac+d^2)(dc-ax)}{dcx(x+d)}$, $\frac{3}{10}$.

**9.** $\frac{\mathrm{R}(y+c)}{\mathrm{R}-(y+c)}$, −2·857, −1·667, −0·9091, 1·111, 3·333, 5·873, 7·648.

**10.** 8·697.

# CHAPTER VIII.

## Exercises XXXV.

**1.** (i) $xc-xd+yc-yd$; (ii) $yp+yq-zp-zq$; (iii) $x^2+x(a-k)-ak$;
(iv) $x^2-a^2$; (v) $x^3+x^2(a+b+c)+x(ab+bc+ca)+abc$;
(vi) $x^3-x^2(l+m+n)+x(lm+mn+nl)-lmn$.

**2.** (i) $4a^2-12ab+9b^2$; (ii) $9p^2+30pq+25q^2$; (iii) $4a^2-9b^2$;
(iv) $25p^2-16k^2$; (v) 999951; (vi) $a^2+b^2+c^2+2ab+2bc+2ca$.

**3.** (i) $a(a+3b)$; (ii) $a(a+6c)$; (iii) $3a(a-3b)$; (iv) $p(p-4q+r)$; (v) $(a+b)(x-y)$; (vi) $(x+p)(x+q)$; (vii) $(2x-5z)(3x+y)$; (viii) $(a+b)(c+d)$.

**4.** (i) 999996; (ii) 1·0201; (iii) 8·821; (iv) 0·9994; (v) 10000; (vi) 1·010.

**5.** (i) $(3h-4k)(3h+4k)$; (ii) $(3a-c)^2$; (iii) $1\times 193$; (iv) $2\times 0{\cdot}06$; (v) $(5a+b)^2$; (vi) $(x-2{\cdot}236)(x+2{\cdot}236)$; (vii) $(2x+2{\cdot}646)(2x-2{\cdot}646)$; (viii) $(1{\cdot}005x+1{\cdot}265)(1{\cdot}005x-1{\cdot}265)$.

**6.** (i) $(3x-4a)(x+2a)$; (ii) $(a-2b)(a+2b)$; (iii) $(a-b+c)(a+b-c)$; (iv) $(a-2b)(a-3c)$; (v) $(a-b)(a+b+c)$.

## Exercises XXXVI.

**1.** (i) $x^2+5{\cdot}2x+6{\cdot}4$; (ii) $x^2-1{\cdot}8x-13{\cdot}63$; (iii) $x^2-x(a+1{\cdot}6)+1{\cdot}6a$; (iv) $x^2-x(11{\cdot}7+4{\cdot}3k)+50{\cdot}31k$; (v) $x^2+x(k-1{\cdot}2l)-1{\cdot}2kl$; (vi) $a^2x^2-1{\cdot}6akx-4{\cdot}65k^2$.

**2.** (i) $(a+3)(a+2)$; (ii) $(x-4)(x-3)$; (iii) $(p+8)(p-5)$; (iv) $(a+4)(a-2)$; (v) $(x+12)(x-10)$; (vi) $(k-13)(k+5)$.

**3.** (i) $(x+5)(x-4)$; (ii) $2(a-2b)(a+6b)$; (iii) $(x-3)(x+3)(x-2)(x+2)$; (iv) $a\left(a-\frac{1}{3}\right)\left(a+\frac{1}{4}\right)$.

**4.** (i) $(x+0{\cdot}5)(x+0{\cdot}8)$; (ii) $(x+0{\cdot}8)(x-0{\cdot}5)$; (iii) $(x-0{\cdot}8)(x+0{\cdot}5)$.

**5.** (i) $(a-0{\cdot}25)(a+1)$; (ii) $(a+1)(a+0{\cdot}25)$; (iii) $(a-1)(a+0{\cdot}25)$.

**6.** (i) $(x-3{\cdot}4)(x+1{\cdot}5)$; (ii) $(x+3{\cdot}4)(x-1{\cdot}5)$; (iii) $(x-3{\cdot}4)(x-1{\cdot}5)$.

**7.** (i) $(5-x)(13-x)$; (ii) $(xy-11)(xy-9)$; (iii) $(12+pq)(11-pq)$; (iv) $(x+15y)(x-3y)$; (v) $(x+13y)(x-10y)$.

## CHAPTER IX.

## Exercises XXXVII.

**1.** 18·8244; 4·40.

**2.** 3·142.

**3.** £1. 19*s.* 2*d.*

**4.** 43,191,000.

**5.** $\dfrac{m-bc}{n-b}$ lb.

**6.** $\dfrac{pa+qb}{p+q}$, 26·5 per thousand.

**7.** 11 stone 8 lb.

**8.** 4 days 2 hr.

## Exercises XXXVIII.

**1.** 67·0 % men, 26·3 % women, 6·74 % children.

**2.** 3·70 % copper, 7·41 % antimony, 88·89 % tin; 5·56 lb., 11·11 lb., 133·34 lb.

**3.** 39·2 % tin, 60·8 % lead. **4.** 68·9 % copper, 19·6 % zinc, 6·3 % lead.

**5.** 54·03 % men, 39·31 % women, 6·67 % children.

**6.** 75·18 % carbon, 6·97 % hydrogen, 5·91 % oxygen, 4·09 % nitrogen, 7·86 % ash.

## Exercises XXXIX.

**1.** 0·3. **2.** $\frac{4}{3}$. **3.** 0·24. **4.** 8. **5.** 3·733. **6.** 3·174.
**7.** 1·364. **8.** 90. **9.** 2·4. **10.** 0·625. **11.** 26. **12.** 0·0168.

## Exercises XL.

**1.** $y=1{\cdot}75\sqrt{x}$; 8·75.

**2.** (*a*) Yes; price for 3 tons £3. 6*s*. 0*d*. in each case. (*b*) No; area in second case is 4 times first. (*c*) No; but interest is.

**3.** 6 ft. 7·41 in. **4.** 1 gallon = 10·02 lb.

**5.** 981 cm. per sec. **6.** 0·1034 Kg. per sq. cm.

## Exercises XLI.

**1.** Make each vertical division read 2 cm.; each horizontal division 1 in., *i.e.* multiply scale by 10.

**2.** 0·615, 1·95, 4·56, 3·17, 25·4, 45·1. **3.** 36·7, 73·0, 10·9.

**4.** 5·65 in., 2·61 in. **5.** Circ. = 3·142 × diam.

**6.** (*a*) 14*s*. 1*d*., (*b*) 10*s*. 3*d*. (*c*) £2. 11*s*. 0*d*.; (*a*) 6·50$, (*b*) 2·43$, (*c*) 1·02$, (*d*) 25.67$.

**7.** The last, then the first, then the second. **8.** 1903, 1901.

**9.** 0·2 mile from C at 8.40 a.m. **10.** C, then A, then B.

**11.** Starts 7.23 p.m.; 45 miles per hour.

**12.** Up to £570; between £600 and £640; between £700 and £740.

## CHAPTER X.

## Exercises XLII.

**1.** 6010 sq. yd. **2.** 14·14 c.c. **3.** 1697 lb. **4.** 51·96 c.c.

**5.** 22 ft./sec.; 15 m./hr. **6.** (*a*) $\frac{\pi d^2 l}{4}$; (*b*) 96 c.c. **7.** 2890 gm.

**8.** 6·928 in. **9.** (*a*) 226·2 c.c.; (*b*) 4021 gm.

**10.** $W=\pi lw(R^2-r^2)$. **11.** 1825 gm. **12.** 117200 gm.

**13.** (*a*) 4400; (*b*) 7240; (*c*) 126.

**14.** 63·613 c. in., 0·2294 gall., 12·86 lb. **15.** 6·325 in., 39·74 sq. in.

**16.** 3·767 lb. **17.** $\frac{\pi}{4}t(D^2-d^2)$. **18.** 0·6667 in.

## PRELIMINARY CERTIFICATE EXAMINATION. (M.C.U.)

### TECHNICAL GROUP. PART I.

### Practical Mathematics.

**1.** (*b*) 38°, 89°, 53°; (*c*) 10·76 sq. in.
**2.** 18·01786, 0·01801786, 18017·86.
**3.** (*a*) 7·194 in.; (*c*) 89·4 sq. in.; (*d*) 92·3 cub. in. **4.** 16*s.* 6*d.*
**5.** (*a*) 3; (*b*) −10·5; (*c*) 3. **6.** (i) −3; (ii) 0; (iii) $-\frac{2}{15}$; (iv) $\frac{8}{9}$.
**7.** (*a*) $4x^2-9$; (*b*) $x^2+6x-16$; (*c*) $a^4-b^4$; (*d*) $x+7$. **8.** 38.
**9.** (*a*) $V=t(\pi R^2-l^2)$; (*b*) 8·18 cub. in.

## PRELIMINARY TECHNICAL COURSE. (U.L.C.I.)

### FIRST YEAR.

**1.** (*a*) $8\frac{11}{14}$; (*b*) 30. **2.** $1\frac{19}{32}$. **3.** (*a*) 100; (*b*) 1000; (*c*) 0·1.
**4.** 4·10″, 3·54″, 2·87″, 2·24″, 10·16 sq. in. **5.** 0·6″, 1·8″, 4·2″.
**6.** (*a*) 5; (*b*) 12. **7.** 5·445 sq. in. **8.** $x=11$, $y=4$.
**9.** (i) 10; (ii) 14. **10.** 4 ft. 2 in., 10 in.

### SECOND YEAR.

**1.** (*a*) 27; (*b*) 26·8; (*c*) 26·83, 0·64%. **2.** 3·142.
**3.** 196430. **4.** 7 ft.
**5.** $x^2+2xy+y^2$, $x^2-2xy+y^2$, $a^2-4ab+4b^2$, $(x^2+4y^2)(x+2y)(x-2y)$, $(a-4b)(a-3b)$.
**6.** 910 sq. ft. **7.** $2x^2-3xy-2y^2$, $4·5\times 1$. **8.** (*a*) 5; (*b*) 4.
**9.** 5 ft. 5·368 in. **10.** 3·15 in.

# INDEX

ABBREVIATIONS, 13
Addition, 1, 35, 123
Algebraic graph, 100
Archimedes, 73
Area, 54, 158
Average, 138
Axis, 54, 159

BRACKET, 8, 82

CALCULATION, 34
Coefficient, 134
Cone, 159
Constant, 146
Contour, 5
Cube, 47
Cuboid, 67
Cylinder, 158

DECIMALS, 17
Denominator, 118
Density, 29
Difference, 3
Division, 10, 24, 44, 47, 124

EQUATIONS, 76
Expansions, 128
Expression, 1

FACTOR, 113, 134
Factorisation, 129
Formula, 72
Fraction, 118

GENERALISED ARITHMETIC, 40
Generator, 158
Graph, 87
  „ (Magnification of), 97
  „ (Types of), 99

H.C.F., 114

INDEX, 20, 22
Inverse, 3, 142

L.C.M., 116

MENSURATION, 54
Metric system, 26
Minus, 3
Multiplication, 7, 24, 41, 47, 124

NEGATIVE, 5
Numerator, 118

PERCENTAGE, 141
$\pi$, 73, 158
Plus, 1
Positive, 5
Powers, 20, 47
Prime, 113
Problems, 80
Product, 7
Proportion, 145

QUOTIENT, 11

RATE, 98
Ratio, 143
Reciprocal, 75
Rectangle, 54, 57

SHEAR, 58
Significant figures, 32
Signs, 13
Similar triangles, 150
Slope, 153
Specific gravity, 30
Speed, 153
Sphere, 158
Square, 47, 57
Square root, 48
Squared paper, 89
Straight line graph, 150
Stress, 16
Subtraction, 3, 12, 38, 123
Sum, 2

TABLES, 86
Theorem of Pythagoras, 60
Triangle, 57

VARIABLES, 146
Variation, 147
Volume, 67, 158

WEIGHT, 29

GLASGOW: PRINTED AT THE UNIVERSITY PRESS BY ROBERT MACLEHOSE AND CO. LTD.

Mat
U855f

145937

Author Usherwood, T. S. and Trimble, C. J. A.

Title A first book of practical mathematics.

University of Toronto
Library

DO NOT
REMOVE
THE
CARD
FROM
THIS
POCKET

Acme Library Card Pocket
Under Pat. "Ref. Index File"
Made by LIBRARY BUREAU